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Optimization of kinematic and geometric parameters in three-element grinding discs with a central rotational axis for the uniformity of concrete surface treatment

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ABSTRACT:
The final stage of grinding concrete surfaces is performed by floaters with solid grinding discs. The working element in the shape of a solid wheel uses the full surface of the disc, ensuring maximum geometrical effectiveness. The disc moves in a steady progressive manner, while at the same time, rotating around its center axis. The contact line length of the full disc with the machined surface is set by geometrical effectiveness S_g. After one cycle of machining the surface with the full disc, the geometrical effectiveness measured at points on a line in a perpendicular direction from the disc moving direction has a local minimum within the machining area and zero values on the edges and outside of the machined area. Decreasing the unfavorable impact of local minimum effectiveness inside the central machining area for the uniformity of machining can be achieved by using discs with concentric working elements – rings and wheel with correctly selected sizes and rotation speeds. For increasing the machining uniformity at the edges of the machined area, partial overlap of machining zones is used in the following cycles. The article describes a disc in a three-element system consisting of a wheel, and two rings used for optimizing uniformity of machining concrete surfaces.

KEYWORDS:
geometric effectiveness; floating; concrete; optimalization

1. Introduction

Concrete surface treatment has a significant impact on reducing defects and increasing the durability of concrete floors. Trowels are commonly used for troweling concrete surfaces due to their high efficiency, simple construction and high reliability [1, 2]. Finishing mashing, which is the final stage of surface treatment, is carried out with a solid wheel with a circular working element. Often, the process involves rubbing in sprinkles to refine the surface. The length of the contact line of the full disc with the point of the machined surface is determined by a parameter called the geometric efficiency S_g.

The blurring disk moves in a uniform, straight, translational motion while simultaneously rotating around its axis at a constant angular velocity. The geometric efficiency of S_g at the midpoints of the machining zone reaches a local minimum which adversely affects the uniformity of machining. The paper examines the possibility of reducing the unevenness of the interaction using a disk consisting of a circle in the middle of the disk and two concentric rings. Individual working elements can adopt different angular speeds. It is assumed that increasing the angular velocities of the central working element will increase the value of the geometric efficiency in the central part of the disk locally, increasing the machining uniformity of the entire system.

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This was confirmed for a two-element disc in [3]. The task set out in the work is the optimal selection of the dimensions of the wheel and middle ring elements and their rotational speeds giving the best uniformity of surface treatment at the assumed value of geometric efficiency.

2. Surface treatment with a full disc

The amount of work put into mashing the surface at a given point can be compared using the geometric efficiency parameter S_g. The geometric efficiency S_g at the point of the surface being treated is defined as the length of the contact line of the surface of the blurring disc with the point of the surface being treated [4].

The solid disc has one wheel-shaped working element that fills the entire surface of the disc. Model-wise, it moves in a uniform, straight motion while simultaneously rotating around an axis in the center of the circle with a uniform angular velocity. The area of impact created this movement is a rectangle with the width of the disc diameter and its axis of symmetry drawn by the center of the disc. At that moment, the disc acts on the surface with which the disc is in contact, i.e. in this case, in the shape of a circle.

As a result of the translational movement of the disc in the area of its impact, the speed vectors at all points of the disc with the points of the machined surface have the same value, the directions are parallel and have the same turns in accordance with the assumed parameters of the movement of the disc center.

The linear speed vector at the analyzed point resulting from the rotation of the disc has a value directly proportional to the rotational speed of the disc ω and to the length of the radius defined by the center of the disc and the analyzed point, and the direction of the vector is perpendicular to the radius, and its turn depends on the direction of rotation.

The speed of the V_w target at any point is the sum of the vector of the forward speed of the disc and the linear speed resulting from the rotational speed. The diagram for determining the resultant velocity vector for a point in the area of the disk impact is shown in Figure 1.

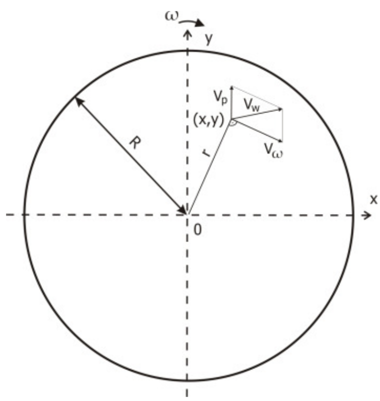

Fig. 1. Scheme for determining the resultant velocity vector resulting from the forward and rotational speed of a disc for any point with coordinates relative to the center of the disc

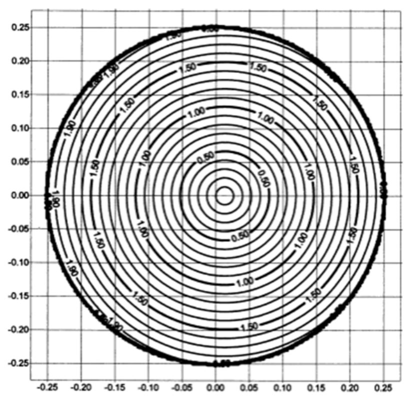

Fig. 2. Distribution of the impact speed on the surface of the full disc D = 0.5 m, V_p = 0.1 m/s, ω = –7.54 rad/s

The resultant velocity vector module V_w of the disc moving with translational velocity V_p and rotational speed ω at point P with coordinates x, y in the coordinate system with the origin in the center of the disc and the Y axis parallel to the translational velocity vector V_p as in Figure 1 is given by:

$$V_w(x,y) = \sqrt{V_p^2 + 2V_p x\omega + y^2\omega^2 + x^2\omega^2} \tag{1}$$

The speed distribution for the full disc is shown in Figure 2.
The geometric efficiency S_g at a given point P(x, y) of the machined surface after the complete passage of the target through the tested point is the sum of the products of the accidental velocity of the impact V_w and the duration of the impact t as determined by the formula:

$$S_g(x) = \int_{t_p}^{t_k} V_w\,(x,t)dt \tag{2}$$

where: t_p – start time of the point of contact with the target, t_k – end of contact time with the target, x – abscissa of the examined point in the coordinate system with the center in the center of the dial as in Figure 1. Taking into account the constant progressive speed and the length of the vertical chord of the wheel for the x-ordinate, assuming $t = 0$ for the position of the center of the disc, the values of t_p and t_k can be calculated depending on the x-ordinate. After integration, we obtain a formula determining the geometric effectiveness of the impact of the full disc on the points of the machined surface depending on the value cut off relative to the center of the disc:

$$S_g(x) = \frac{1}{V_p}\sqrt{(R^2 - x^2)\left(V_p^2 + 2V_p\omega x + \omega^2R^2\right)} + \\ + \frac{\left(V_p + x\omega\right)^2}{2\omega V_p}\ln\left(\frac{\sqrt{V_p^2 + 2V_p\omega x + \omega^2R^2} + \omega\sqrt{R^2 - x^2}}{\sqrt{V_p^2 + 2V_p\omega x + \omega^2R^2} - \omega\sqrt{R^2 - x^2}}\right) \tag{3}$$

The above formula (3) makes sense for $x \notin (-R; R)$, $V_p > 0$ and $\omega \neq 0$.
Other cases can be easily determined.
For $x \notin (-R; R)$ the point is outside the area of impact of the disc, therefore the value of geometric efficiency is $S_g = 0$. For zero forward speed $V_p = 0$, the disc stands still, so the value of geometric efficiency is infinitely large, which has no practical application. For rotational speed $\omega = 0$ the value of geometric efficiency is the length of the vertical chord of the wheel passing through the abscissa x determined by the formula:

$$S_g(x) = 2\sqrt{R^2 - x^2} \tag{4}$$

Details regarding the removal of the above patterns are provided in [5].

3. Geometric efficiency of the ring element

The geometrical effectiveness of the interaction of an annular element with an outer radius R_z and an inner radius R_w after a single disk transition can be calculated using the superposition principle by subtracting the efficiency of the wheel with radius R_w from the efficiency of the radius R_w as described by the formula:

$$S_g = S_{gRz} - S_{gRw}$$

where: S_{gRz} – geometrical efficiency calculated for a circle with a radius of R_z, S_{gRw} – geometrical efficiency calculated for a circle with a radius of R_w.

After substitutions and simplification, the formulas will take the following form for individual cases:
for $|x| \in (0; R_w)$

$$S_g(x) = \frac{1}{V_p}\left(\sqrt{(R_z^2 - x^2)(V_p^2 + 2V_p\omega x + \omega^2 R_z^2)} - \sqrt{(R_w^2 - x^2)(V_p^2 + 2V_p\omega x + \omega^2 R_w^2)}\right) + \\ + \frac{(V_p + x\omega)^2}{\omega V_p} \ln\left(\frac{\sqrt{V_p^2 + 2V_p\omega x + \omega^2 R_z^2} + \omega\sqrt{R_z^2 - x^2}}{\sqrt{V_p^2 + 2V_p\omega x + \omega^2 R_w^2} + \omega\sqrt{R_w^2 - x^2}}\right) \tag{5}$$

for $|x| \in (R_w; R_z)$ the formula takes the same form as for a circle

$$S_g = \frac{1}{V_p}\sqrt{(R_z^2 - x^2)(V_p^2 + 2V_p\omega x + \omega^2 R_z^2)} + \\ + \frac{(V_p + x\omega)^2}{2\omega V_p} \ln\left(\frac{\sqrt{V_p^2 + 2V_p\omega x + \omega^2 R_z^2} + \omega\sqrt{R_z^2 - x^2}}{\sqrt{V_p^2 + 2V_p\omega x + \omega^2 R_z^2} - \omega\sqrt{R_z^2 - x^2}}\right) \tag{6}$$

for $|x| > R_z$ the geometric efficiency value is $S_g = 0$.

4. Geometric efficiency of complex ring-circular systems

Systems consisting of concentric annular and circular working elements with a common center of rotation arranged in such a way as to exclude the existence of common parts of these surfaces can be calculated from the principle of superposition by summing their interactions.

The value of geometric efficiency can be written by the general formula:

$$S_g(x) = \sum_{i=1}^{n} S_{gi}(x) \tag{7}$$

5. Assessment of the uniformity of the disc impact

The uniformity of the impact is determined by the standard deviation index ε assuming a minimum value of zero for a completely uniform impact and increasing the value with an increase in the dispersion of values for the test sample.

From a mathematical point of view, the goal is to minimize the standard deviation index of geometric efficiency ε. The standard deviation index for the geometric efficiency S_{gi} calculated in n points along the length of the measuring segment spaced at equal distances from each other is described by the formula:

$$\varepsilon = \frac{\sigma}{\overline{S_g}} = \sqrt{\frac{\frac{1}{2}\left(S_{g1} - \overline{S_g}\right)^2 + \sum_{i=2}^{n-1}\left(S_{gi} - \overline{S_g}\right)^2 + \frac{1}{2}\left(S_{gn} - \overline{S_g}\right)^2}{\overline{S_g}^2 (n-1)}} \tag{8}$$

6. Characteristics of the distribution of the geometric efficiency value S_g for a full disc

With a single pass of the full disc, the distribution of effectiveness S_g in a direction perpendicular to the direction of the disc is presented as in Figure 3. To increase uniformity, the pattern of disc movement is used for surface treatment with partial overlap shown in Figure 5. The use of optimal overlays increases the uniformity of machining surface as shown in Figure 4.

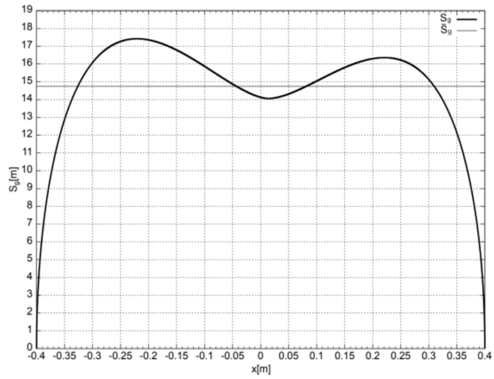

Fig. 3. Graph of geometric efficiency S_g for a single full face transition D = 0.8 m, V_p = 0.1 m/s, ω = –8,8 rad/s, S_g = 14.7475 m, ε = 0,184 m

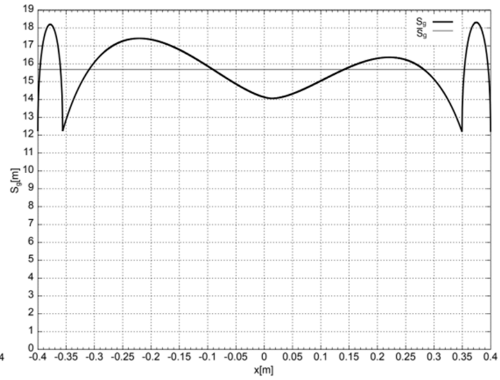

Fig. 4. Graph of geometric efficiency S_g for a full disc with optimally applied machining zones D = 0.6 m, V_p = 0.1 m/s, ω = –8.8 rad/s, a = 0.043718 m, b = 0.504846 m, S_g = 15.6702 m, ε = 0.079 m

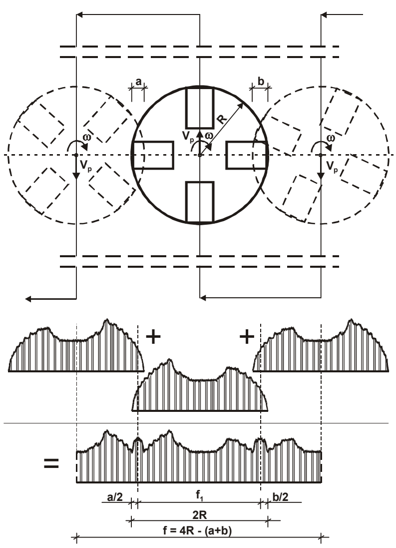

Fig. 5. The principle of using overlays to increase the uniformity of machining for rectilinear movement of the disc [6]

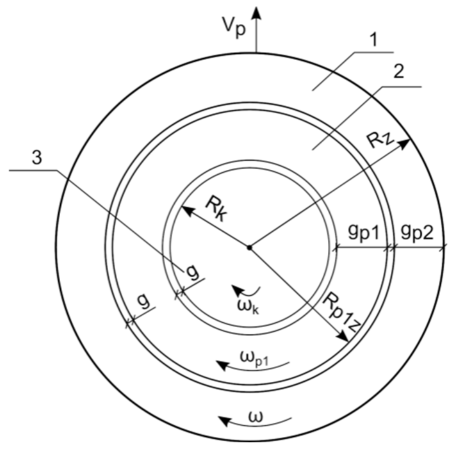

Fig. 6. Diagram of a three-element mashing disc with a central axis of rotation consisting of three working elements outer (1), middle (2) ring and wheel (3) that can move with independent angular speeds

7. Geometry and kinematics of the analyzed three-element disc

Analysis of the shape of the graph of the impact of the full mashing disk shown in Figure 4 indicates the possibility of increasing the uniformity of impact by increasing the machining efficiency in the middle of the graph. This is possible by replacing the homogeneous full disc with a two-piece disc consisting of: a wheel located in the middle of the disc rotation and rings in such a way that they collectively fill the geometry of the full disc as shown in [3]. The three-element geometry analyzed consists of a circle and two concentric rings. All components can rotate at different speeds in any direction.

As a benchmark for improving machining parameters, a trowel with a full disc, consisting of the following parameters, was used:
- progressive speed V_p = 0.1 m/s
- rotation speed ω = –8.8 rad/s (–84 rpm, the disc rotates clockwise)
- disc radius R_z = 0.4 m.

The diagram of the three-element disc is presented in Figure 6. The purpose of the considerations is to check the possibility of creating a geometrical system of a three-element disc with better parameters of machining uniformity compared to a two-element disc under the following conditions:
- the outer radius of the outer ring of the three-element system is the radius of the R_z target,
- the rotational speed of the outer ring is consistent with the rotational speed of the reference disc ω,
- distances between individual working elements of the disc are g = 1 mm,
- rotational speeds and directions of movement of elements can be any, but due to the nature of the impact, the linear speeds of any working element are limited to the maximum speed of the reference disk.

Definitely defining the geometry and kinematics of the system from Figure 6 requires the determination of four additional parameters:
- the central radius of the circular work element R_k,
- the radius of the outer middle ring R_{p1z},
- rotational speed of the circular working element ω_k,
- rotational speed of the middle ring ω_{p1}.

The geometrical relationships of the system are described by the formulas:

$$\begin{gathered} R_{p1w} = R_k + g \\ R_{p2w} = R_{p1z} + g \\ R_z = R_k + g_{p1} + g_{p2} + 2g \end{gathered} \tag{9}$$

where: R_{p1w} – inner radius of the inner ring, R_{p2w} – inner radius of the outer ring, g_{p1} – inner ring thickness, g_{p2} – outer ring thickness, g – distance between working elements.
Due to the variable boundaries of parameter values that must meet the relationships (9), the use of optimization procedures is troublesome [7].

The solution to the problem facilitates the exchange of variables for others that uniquely define the system at constant limits:

w_{rk} – coefficient that determines the radius of the circle to the radius of the disc, which can take values from 0 to 0.95,

w_{gp1} – coefficient of thickness of the middle ring in relation to the remaining space in the dial after taking into account the radius of the central circle R_k and the distance between the elements g, which can take values from 0 to 0.95,

$w_{\omega k}$ – factor determining the rotational speed of the wheel working element in relation to the maximum linear speed, which can take values from –1.0 to 1.0,

$w_{\omega p1}$ – factor determining the rotational speed of the working element of the middle ring in relation to the maximum linear speed, which can take values from –1.0 to 1.0.

The conversion of the above coefficients into physical values can be performed using the following relationships:

$$\begin{gathered} R_k = w_{rk} \cdot R_z \\ g_{p1} = w_{gp1}(R_z - R_k - 2g) \\ g_{p2} = R_z - R_k - g_{p1} - 2g \\ \omega_k = w_{\omega k}\frac{\omega_z \cdot R_z}{R_k} \\ \omega_{p1} = w_{\omega p1}\frac{\omega_z \cdot R_z}{R_{p1}} \end{gathered} \tag{10}$$

The geometrical effectiveness of the S_g interaction of the discs on the machined surface was determined for 1601 points evenly distributed over the measured section with a disc diameter 0.8 m long set perpendicular to the direction of the translational movement of the disc for rectilinear movement with optimal overlays.

The distribution of geometric efficiency for the optimal parameters of a two-element disc is shown in Figure 7, and for a three-element disc is shown in Figure 8. The standard deviation indicator for a three-element disc takes the value ε = 0.061994 m, and the machining uniformity is better than the two-element disc ε = 0.0696972 m and the full disc whose S_g graph is shown in Figure 4 for which ε = 0.079 m. The charts for multi-element discs show an increase in the geometric efficiency in the middle part caused by the operation of a circular element with increased rotational speed.

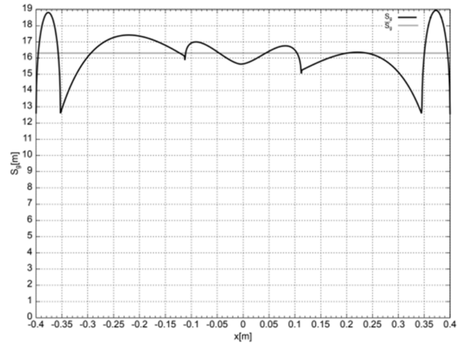

Fig. 7. Graph of geometric efficiency S_g for a two-element disc optimized for uniformity of machining with optimal overlays D = 0.8 m, V_p = 0.1 m/s, ω = –8.8 rad/s, a = 0.049 m, b = 0.052 m, S_g = 16.3081 m, ε = 0.0696972 m [5]

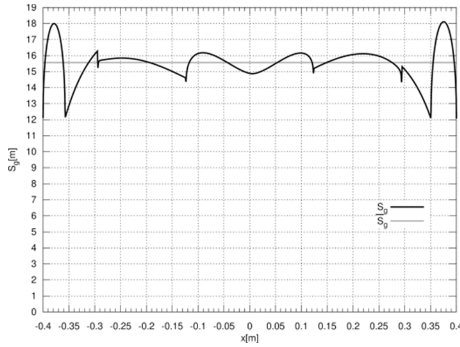

Fig. 8. Graph of geometric efficiency S_g for a three-element disc optimized for uniformity of machining with optimal overlays D = 0.8 m, V_p = 0.1 m/s, ω = –8,8 rad/s, a = 0.0426 m, b = 0.04921 m, S_g = 15.5668 m, ε = 0.061994 m

8. Optimization of disc parameters for processing areas with a width smaller than the diameter of the disc

A diagram of the principle selection of machining parameters for areas with a width smaller than the diameter of the disk is shown in Figure 9. The machining process consists of a single pass of the disk on the surface to be treated. From a mathematical point of view, the movement line of the disc is selected in relation to the machined area so that the segment of the plot with the width of the piece (the hatched part in Fig. 9) has the most even distribution. For this purpose, one more parameter was added to the optimization that determines the position of the center of the disc relative to the machined area during machining. By modifying the parameters, the optimal shape and position of the S_g plot is found for a given machining width. The ability to choose the position of the disc relative to the surface to be treated is large for a small width of the machining area and decreases as it increases. When the width of the machining area is equal to the diameter of the blade, there is only one position in the center of the blade that allows the entire width of the area to be machined.

A climbing algorithm was used to optimize the system, where the objective function was to minimize the standard deviation index. The calculations were made independently for the width of the processed area from l = 0.1 m to the full diameter of the disc 0.8 m in increments of 0.01 m assuming the required minimum average values of geometric efficiency S_g with values 3, 7, 11 and 14 m. The value of 14 m corresponds to the standard full disc.

Figure 10 presents the values of the standard deviation coefficient ε determining the uniformity of the impact distribution for a two-element and full disc, and in Figure 11 the same for a three-element disc. In the whole area of the analyzed machining widths, a three-element disc shows greater uniformity than a full and two-element disc. Figures 12-17 show the values of parameters obtained in the optimization process in all cases presented here.

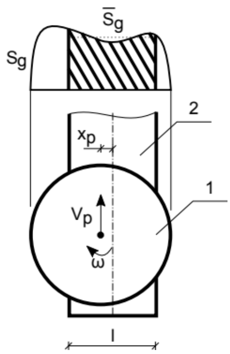

Fig. 9. Schematic of the principle of selecting the most even machining area with a width *l* of a seized linear prefabricated element with a width of the machining area smaller than the diameter of the grinding disk; 1 - scuffing disk, 2 - prefabricated [3]

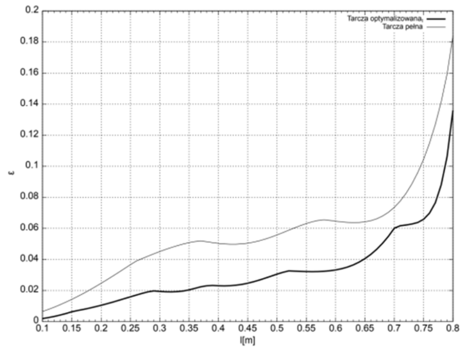

Fig. 10. Graph of standard deviation index *ε* in an optimally selected mashing range depending on the width of the interval *l* for the optimized disc and wheel D = 0.8 m, V_p = 0.1 m/s, ω = –8,8 rad/s [3]

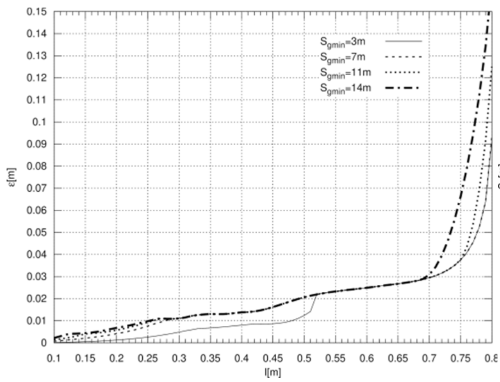

Fig. 11. Graph of standard deviation index ε in an optimally selected mashing range depending on the processing band width *l* and the required minimum S_g value for a three-element disc V_p = 0.1 m/s, ω = –8.8 rad/s

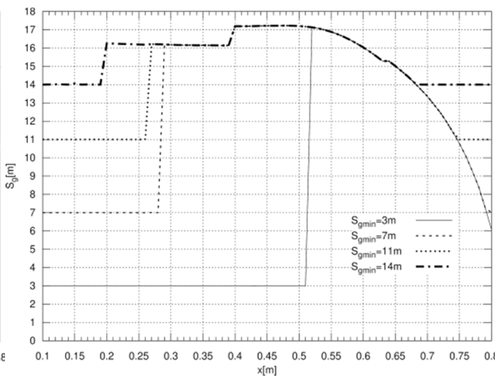

Fig. 12. Graph of the value of the average geometric efficiency S_g of a three-element disc depending on the width of the processing band *l* and the required minimum value S_g D = 0.8 m, V_p = 0.1 m/s, ω = –8,8 rad/s

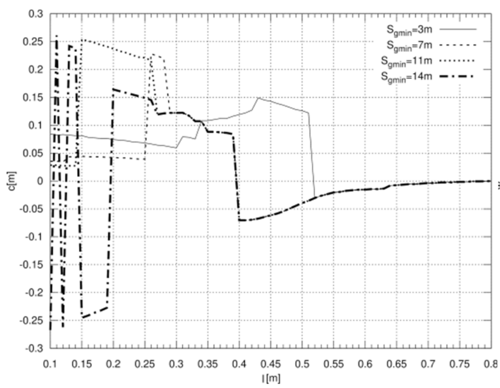

Fig. 13. Diagram of the shift *c* of the position of the three-element disc relative to the center of the machined area depending on the width of the processing band *l* and the required minimum value S_g D = 0.8 m, V_p = 0.1 m/s, ω = –8.8 rad/s

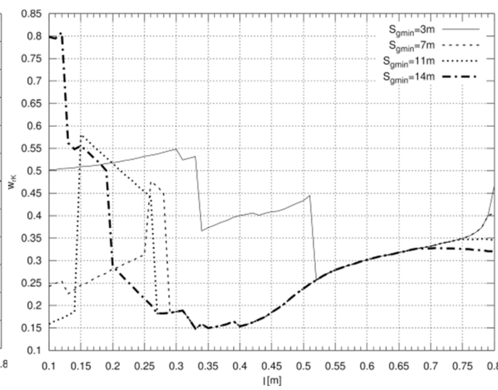

Fig. 14. Graph of the value of the w_{rk} factor determining the ratio of the center radius of the working element to the radius of the disc D = 0.8 m, V_p = 0.1 m/s, ω = –8,8 rad/s

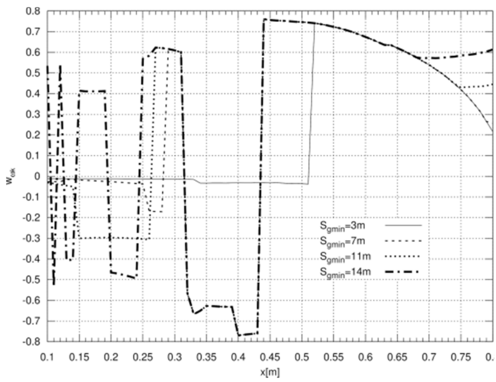

Fig. 15. Graph of the coefficient wωk determining the ratio of the angular velocity of the wheel to the angular velocity of the outer ring. D = 0.8 m, V_p = 0.1 m/s, ω = –8.8 rad/s

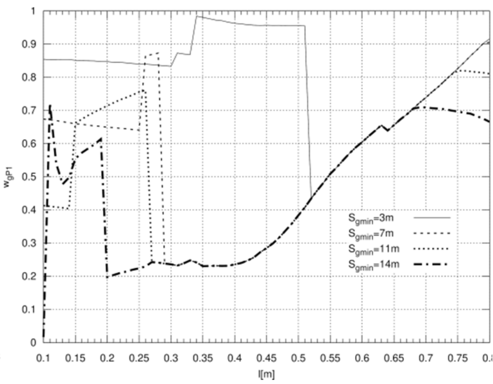

Fig. 16. Graph of the factor w_{gp1} determining the thickness of the middle ring in relation to the remaining space in the disc after taking into account the radius of the central circle R_k and the distance between elements g = 1 mm, D = 0.8 m, V_p = 0,1 m/s, ω = –8.8 rad/s

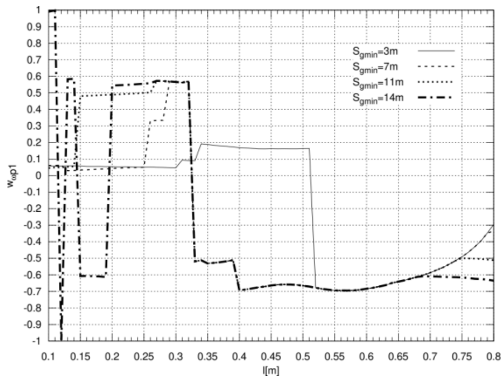

Fig. 17. Graph of the coefficient $w_{\omega p1}$ determining the ratio of the maximum angular velocity of the middle ring to the maximum angular velocity of the outer ring D = 0.8 m, V_p = 0.1 m/s, ω = –8.8 rad/s

9. Results

The use of three-element mashing discs with a central axis of rotation makes it possible to increase the uniformity of surface treatment compared to a full and two-piece disc. For surface treatment with a large width, a trajectory with overlapping machining areas is required. The uniformity of machining increases significantly in the middle zone, however, due to the significant differences in S_g values in the overlay zone, it does not significantly affect the overall improvement in the uniformity of machining expressed by the standard deviation index. Better effect is obtained when machining areas with a width smaller than the diameter of the mashing disc, which can be used for surface treatment of prefabricated elements. The narrower width of the machined surface than the diameter of the disc allows you to select the area with the highest uniformity of machining which significantly increases the uniformity of machining. Optimal selection of geometrical and kinematic parameters increases the uniformity of the three-element disc in relation to the full and two-element discs. The use of a three-element disc with appropriately selected parameters allows you to adapt the geometric effectiveness of the disc to technological requirements.

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Optymalizacja parametrów kinematycznych i geometrycznych tarcz zacierających trójelementowych o centralnej osi obrotu ze względu na równomierność obróbki powierzchni betonowych

STRESZCZENIE:
Finalny etap zacierania powierzchni betonowych wykonuje się zacieraczkami z tarczami pełnymi. Element roboczy o kształcie koła wykorzystuje całą dostępną powierzchnię tarczy, zapewniając jej maksymalną skuteczność geometryczną. Tarcza porusza się ruchem jednostajnym postępowym i jednocześnie obraca wokół swojego środka. Długość linii kontaktu tarczy pełnej z punktem obrabianej powierzchni określa skuteczność geometryczna S_g. Po jednokrotnym cyklu obróbki powierzchni tarczą pełną skuteczność geometryczna mierzona w punktach na linii w kierunku prostopadłym do kierunku ruchu tarczy posiada minimum lokalne w środku i wartości zerowe na krawędzi i poza obszarem oddziaływania. Zmniejszenie niekorzystnego wpływu lokalnego minimum skuteczności geometrycznej w środkowej strefie oddziaływania na równomierność obróbki powierzchni można uzyskać przez zastosowanie tarczy składającej się z współśrodkowych elementów roboczych pierścieni i koła o odpowiednio dobranych wymiarach i prędkościach obrotowych. W celu zwiększenia równomierności obróbki w pobliżu granicy oddziaływania stosuje się częściowe nakładanie stref obróbki w kolejnych cyklach. W artykule przedstawiono model tarczy w układzie trójelementowym składający się z koła i dwóch pierścieni wykorzystywany do optymalizacji równomierności oddziaływania przy obróbce powierzchni betonowej.

SŁOWA KLUCZOWE:
skuteczność geometryczna; zacieranie; beton; optymalizacja