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Modification of the Magnel method in the design of cross-sections of post-tensioned concrete beams

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ABSTRACT:
Designing cross-sections of prestressed beams based on Magnel's graphic method was first introduced in the mid-20th century. The graphic method is very useful for determining the initial prestressing force and its eccentricity action. This method is also used nowadays despite various computer programs accelerating the process of dimensioning prestressed structures. In the article, the author briefly outlined the idea of the Magnel method, and attempted to combine the classical method of adopting optimal dimensions for a cross-section of a post-tensioned concrete I-beam with the graphic method of Magnel to determine the prestressing force and its eccentricity. The purpose of this test is to simplify Magnel's graphic method and use analytical geometry in calculations to precisely determine the optimal prestressing force and its eccentricity. The prestressing forces and their eccentricity were calculated for beams with spans of 12.0 to 36.0 meters and the results are presented in diagrams.

KEYWORDS:
Magnel diagram; post-tensioned concrete beam; prestressed force

1. Introduction

The design of post-tensioned concrete beams consists primarily of the acceptance of structural materials (concrete, reinforcing steel and prestressing steel) and the optimal shape and dimensions of the cross-section of the element. The next step is to adopt the appropriate prestressing force and eccentricity of the action of this force. In the article, the author attempted to combine these two design steps into one optimal solution.

The adoption of the cross-section of the post-tensioned concrete beam depends on the strength considerations of the materials adopted, the design and the technological aspects of the component. The strength characteristics of concrete, reinforcing steel and prestressed steel require safety conditions to be met in the load-bearing and usability limit states, the optimal cross-section of the element ensures the safe operation of the structure during the implementation stage. The cross-section of the element involves the adoption of the appropriate design technology, which, among other factors, facilitates the correct arrangement of reinforcing bars, compression cables, the arrangement and compaction of the concrete mixture, and in the final stage the compression and injection of cable ducts. Typical cross-sectional shapes of pretensioned and post-tensioned concrete beams in reference to [1] are shown in Figure 1. The optimal cross sections for post-tensioned concrete structures are the T sections, I sections, inverted T, and trough, box. The article shows an analysis of a beam with a I-beam section.

Based on the accepted dimensions of the cross-section of the beam, optimal prestressing forces and eccentricity of the beam can be determined by the Magnel method.

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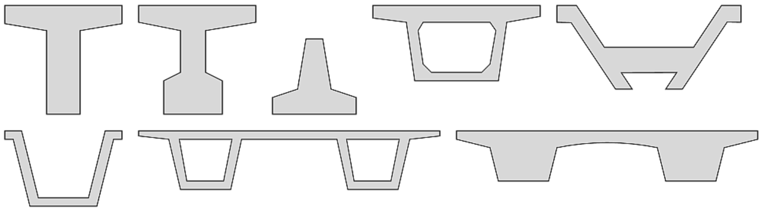


Fig. 1. Examples of shapes of compressed elements [1]

The purpose of modifying the Magnel method is to find the optimal value of the prestressing force and its eccentricity using straight line equations in analytical geometry.

2. Modification of the Magnel method in the design of a prestressed I-beam

The algorithm for determining the compression force and its eccentricity, using the Magnel method, is divided into two stages:
- Stage I – adoption of optimal cross-sectional dimensions of the beam,
- Stage II – determination of the prestressing force and eccentricity of the action of this force.

STAGE I:
Based on position [1] three cases are taken when designing the cross-section of post-tensioned concrete I-beams:
- the case, where the geometrical dimensions of the beam section are imposed from above and on this basis the prestressing force and eccentricity of its operation are determined,
- the case, where the geometrical dimensions of the beam section are given as a function of its height *f*(h) and on this basis the optimum height of that cross-section, the prestressing force and the eccentricity of its operation are determined,
- the case, where the parameters of the generalized cross section are initially determined (using the accepted cross-section geometry coefficients) i.e. cross-sectional area of beam A_c, static moment S_c, moment of inertia J_c, center of gravity of cross-section $S(v_1,v_2)$, the value of the prestressing force and the eccentricity of its action and on this basis the final dimensions of the beams meeting the characteristics of the generalized section shall be selected.

In order to maintain the necessary cross-sectional stiffness of the beam section, the bending strength and the technological considerations of the beam design, cross-sectional factors were used on the basis of [1]. Table 1 shows the cross-factor values for the post-tensioned concrete I-beam. Knowing the values of the coefficients and taking the height of the h cross-section of the beam, you can calculate: the area of the beam section area, the static moment, the moment of inertia, the center of gravity of the cross-section and the bending index for the top and bottom beam fibers. The method of calculating the cross section parameters is given in formulas (1) and (2):

$$A_c = A \cdot h^2 \quad S_c = B \cdot h^3 \quad J_c = C \cdot h^4 \tag{1}$$

$$v_1 = \frac{B}{A} \cdot h \quad v_2 = \frac{A-B}{A} \cdot h \quad W_1 = \frac{A \cdot C}{B} \cdot h^3 \quad W_2 = \frac{A \cdot C}{(A-B)} \cdot h^3 \tag{2}$$

For the analysis, a single-span post-tensioned concrete I-beam with a span of 30.0 meters was used. Using dependency (3), the cross-sectional height of the beam $h = 1.60$ meters was assumed. On this basis, using formulas (1) and (2), the values of the beam section parameters are determined. The final results are given in Table 2.

$$h \cong \langle 0.04\text{–}0.06 \rangle \cdot L_{eff} \Rightarrow h \cong \langle 1.20\text{–}1.80 \rangle \ [\text{m}] \tag{3}$$

Table 1
Coefficients for determining the cross-sectional dimensions of the concrete I-beam

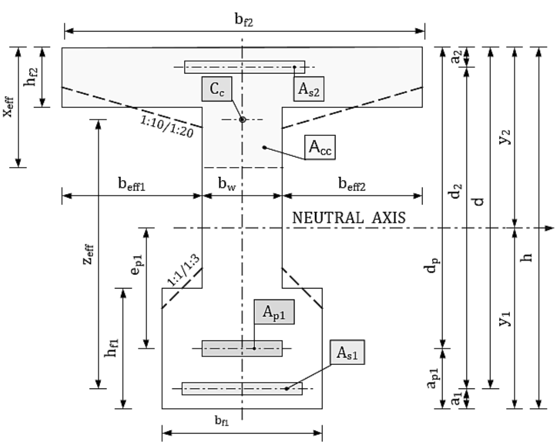
L = (9.0 ÷ 30.0) in [m]	Coefficients for determining the dimensions of the cross-section of a post-tensioned concrete I-beam											Average value
H = (0.04 ÷ 0.06)×L	1	2	3	4	5	6	7	8	9	10	11	
α_{bf1} = ⟨0.30 ÷ 0.60⟩	0.300	0.330	0.360	0.390	0.420	0.450	0.480	0.510	0.540	0.570	0.600	**0.450**
α_{bf2} = ⟨0.40 ÷ 0.80⟩	0.400	0.440	0.480	0.520	0.560	0.600	0.640	0.680	0.720	0.760	0.800	**0.600**
α_{bw} = ⟨0.10 ÷ 0.12⟩	0.100	0.102	0.104	0.106	0.108	0.110	0.112	0.114	0.116	0.118	0.120	**0.110**
α_{hf1} = ⟨0.12 ÷ 0.20⟩	0.120	0.128	0.136	0.144	0.152	0.160	0.168	0.176	0.184	0.192	0.200	**0.160**
α_{hf2} = ⟨0.10 ÷ 0.15⟩	0.100	0.105	0.110	0.115	0.120	0.125	0.130	0.135	0.140	0.145	0.150	**0.125**
A = δ_{Ac} =	0.1540	0.1667	0.1802	0.1945	0.2097	0.2257	0.2425	0.2601	0.2786	0.2979	0.3180	**0.2257**
B = δ_{Sc} =	0.0799	0.0865	0.0935	0.1008	0.1086	0.1168	0.1254	0.2601	0.2786	0.2979	0.3180	**0.1168**
C = δ_{Jc} =	0.0191	0.0212	0.0234	0.0257	0.0281	0.0306	0.0332	0.0358	0.0386	0.0414	0.0442	**0.0306**
δ_{v1} =	0.5191	0.5189	0.5187	0.5183	0.5179	0.5175	0.5171	0.5167	0.5163	0.5159	0.5156	**0.5175**
δ_{v2} =	0.4809	0.4811	0.4813	0.4817	0.4821	0.4825	0.4829	0.4833	0.4837	0.4841	0.4844	**0.4825**
δ_{W1} =	0.0367	0.0408	0.0451	0.0496	0.0543	0.0592	0.0642	0.0694	0.0747	0.0802	0.0858	**0.0592**
δ_{W2} =	0.0396	0.0440	0.0486	0.0534	0.0583	0.0634	0.0687	0.0742	0.0797	0.0854	0.0913	**0.0634**

Table 2
Dimensions and parameters of the cross-section of an I-beam concrete beam

Dimensions of the cross-section of apost-tensioned concrete I-beam		
L = 30.0 m	h = 1.60 m	δ_{Ac} = 0.2257 ⇒ $A_c = \delta_{Ac}\times h^2$ = 0.5778 [m²]
α_{bf1} = 0.450 ⇒	$b_{f1} = \alpha_{bf1}\times h$ = 0.72 [m]	δ_{Sc} = 0.1168 ⇒ $S_c = \delta_{Sc}\times h^3$ = 0.4784 [m³]
α_{bf2} = 0.600 ⇒	$b_{f2} = \alpha_{bf2}\times h$ = 0.96 [m]	δ_{Jc} = 0.0306 ⇒ $J_c = \delta_{Jc}\times h^4$ = 0.20054 [m⁴]
α_{bw} = 0.110 ⇒	$b_w = \alpha_{bw}\times h$ = 0.18 [m]	δ_{v1} = 0.5175 ⇒ $v_1 = \delta_{v1}\times h$ = 0.8280 [m]
α_{hf1} = 0.160 ⇒	$h_{f1} = \alpha_{hf1}\times h$ = 0.26 [m]	δ_{v2} = 0.4825 ⇒ $v_2 = \delta_{v2}\times h$ = 0.7720 [m]
α_{hf2} = 0.125 ⇒	$h_{f2} = \alpha_{hf2}\times h$ = 0.20 [m]	δ_{W1} = 0.0592 ⇒ $W_1 = \delta_{W1}\times h^3$ = 0.2425 [m³]
		δ_{W2} = 0.0634 ⇒ $W_2 = \delta_{W2}\times h^3$ = 0.2597 [m³]

STAGE II:

On the basis of [2-4] and the adopted STAGE I data, two stages of loads of the post-tensioned concrete I-beams were established – the initial stage (initial situation) and the useful stage (permanent situation) in accordance with [5-7]. The characteristic dead load G_{k2} = 10.0 kN/m and the characteristic live load Q_{k1} = 20.0 kN/m were assumed.



A_{cc} – area of concrete compression zone
A_{s1} – area of tension reinforcing steel
A_{s2} – area of compression reinforcing steel
A_{p1} – area of prestressing steel
C_c – center of gravity of concrete compression zone
y_1 – distance of extreme bottom fibres from neutral axis
y_2 – distance of extreme top fibres from neutral axis
e_{p1} – eccentricity of the prestressing steel

Fig. 2. Designations for the cross-section of post-tensioned concrete I-beam

At the initial stage, the bending moment was assumed at M_G = 1643.63 kNm while in the utility stage M_Q = 5018.63 kNm. The cross-section of the beam is shown in (Fig. 2).

The Magnel method was used to determine the optimal value of the prestressing force and its eccentricity. The classic solution of this method is to find a common area of four semi-flats, determined graphically by means of linear equations.

There are four basic conditions associated with stress in the beam stress fibers. The two conditions refer to the initial stage for the bending moment M_G (the characteristic moment of the dead loads) and two conditions for the usable stage for the moment M_Q (the characteristic moment of live loads).

The stress conditions for determining the planes limiting the area according to Magnel are:

$$\sigma_{c1}^{sp} = \frac{P_0}{A_c} + \frac{P_0 \cdot e_{p1}}{W_1} - \frac{M_G}{W_1} \leq 0.6 \cdot f_{ck} \tag{4}$$

$$\sigma_{c2}^{sp} = \frac{P_0}{A_c} - \frac{P_0 \cdot e_{p1}}{W_2} + \frac{M_G}{W_2} \geq -f_{ctm} \tag{5}$$

$$\sigma_{c1}^{su} = \frac{\vartheta \cdot P_0}{A_c} + \frac{\vartheta \cdot P_0 \cdot e_{p1}}{W_1} - \frac{M_Q}{W_1} \geq -f_{ctm} \tag{6}$$

$$\sigma_{c2}^{u} = \frac{\vartheta \cdot P_0}{A_c} - \frac{\vartheta \cdot P_0 \cdot e_{p1}}{W_2} + \frac{M_Q}{W_2} \leq 0.45 \cdot f_{ck} \tag{7}$$

Linear equations for determining an area in the Magnel method:

$$\frac{1}{P_0} \geq \frac{1}{0.6 \cdot f_{ck} \cdot W_1 + M_G} \cdot e_{p1} + \frac{W_1}{A_c \cdot (0.6 \cdot f_{ck} \cdot W_1 + M_G)} \tag{8}$$

$$\frac{1}{P_0} \geq \frac{1}{f_{ctm} \cdot W_2 + M_G} \cdot e_{p1} - \frac{W_2}{A_c \cdot (f_{ctm} \cdot W_2 + M_G)} \tag{9}$$

$$\frac{1}{P_0} \leq \frac{\vartheta}{M_Q - f_{ctm} \cdot W_1} \cdot e_{p1} + \frac{\vartheta \cdot W_1}{A_c \cdot \left(M_Q - f_{ctm} \cdot W_1\right)} \tag{10}$$

$$\frac{1}{P_0} \geq \frac{\vartheta}{M_Q - 0.45 \cdot f_{ck} \cdot W_2} \cdot e_{p1} - \frac{\vartheta \cdot W_1}{A_c \cdot \left(M_Q - 0.45 \cdot f_{ck} \cdot W_2\right)} \tag{11}$$

$$\vartheta = (1 - \alpha_1) \cdot (1 - \alpha_2) \tag{12}$$

where $\alpha_1 \in \langle 0.10; 0.12 \rangle$ and $\alpha_2 \in \langle 0.10; 0.18 \rangle$. Values adopted $\alpha_1 = 0.10$ and $\alpha_2 = 0.12$. Figure 3 shows the final solution of Magnel's modified graphical method.

In the classic Magnel method, the determination of the prestressing force and its eccentricity action ends with the EFGH field being found, which is the acceptable area of the solution created by inequalities (8), (9), (10), (11) and (12). The maximum force value P_t is at point *H*, i.e. at the point of intersection of the straights (8) and (11). The maximum economic prestress force occurs at the intersection of the straight (10) and (11) at point *E*.

The purpose of the Magnel modification presented in the article is to find one optimal solution, i.e. to find the best solution; point $S(e_{p1}; 1/P_t)$ in Figure 3, which is formed from the intersection of optimal straight line *EG* and optimal straight line FH. You can use analytical geometry to write general *EG* and *FH* line equations:

$$EG\colon\ E(X_E; Y_E)\ ;\ G(X_G; Y_G)\ \Rightarrow\ \frac{1}{P_t} - Y_E = \left(\frac{Y_G - Y_E}{X_G - X_E}\right) \cdot \left(e_{p1} - X_E\right) \tag{13}$$

$$FH\colon\ F(X_F; Y_F)\ ;\ H(X_H; Y_H)\ \Rightarrow\ \frac{1}{P_t} - Y_F = \left(\frac{Y_H - Y_F}{X_H - X_F}\right) \cdot \left(e_{p1} - X_F\right) \tag{14}$$

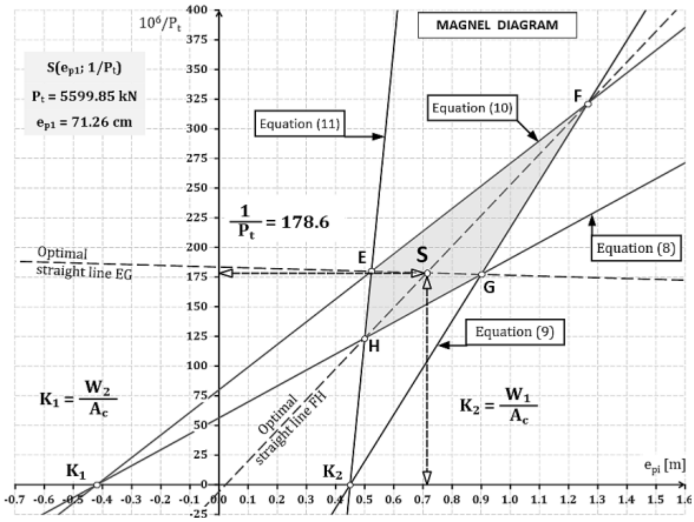


Fig. 3. The final graphic solution of the modified Magnel method

Transforming equations (13) and (14) into an equation system (15) can be determined using the formulas (16) the coordinates of the $S(e_{p1}; 1/P_t)$:

$$\begin{cases} A_1 \cdot e_{p1} + B_1 \cdot \left(\frac{1}{P_t}\right) + C_1 = 0 \\ A_2 \cdot e_{p1} + B_2 \cdot \left(\frac{1}{P_t}\right) + C_2 = 0 \end{cases} \tag{15}$$

$$e_{p1} = \frac{-C_1 \cdot B_2 + B_1 \cdot C_2}{A_1 \cdot B_2 - A_2 \cdot B_1} \quad \frac{1}{P_t} = \frac{-A_1 \cdot C_2 + A_2 \cdot C_1}{A_1 \cdot B_2 - A_2 \cdot B_1} \tag{16}$$

The coordinates of point S are the optimal values for the inverse of the prestressing force after all losses ($1/P_t$) and the eccentricity of the force. The initial prestressing force P_0 should be calculated from the formula (17):

$$P_0 = \frac{P_t}{\vartheta} \tag{17}$$

3. Practical application of the modified Magnel method

Using the modified Magnel method to determine the initial prestressing force of the post-tensioned concrete I-beams, can show in the diagram the values of the prestressing forces after all losses, the initial prestressing forces, and the eccentricity action of those forces as a function of the cross-section height and beam span. The author of the paper presented such a solution on (Fig. 4). Assuming a beam span between 12.0 and 36.0 meters (a change in length of 3.0 meters is assumed), the corresponding optimal instability of the prestressing force and the value of this force both initial and after all losses can be read. For the length of intermediate spans e.g. 19.0 meters, the values are read using linear appreciation between adjacent graphs. Knowing the value of the compression force after all losses, you can quickly calculate the necessary transverse field of the prestressed reinforcement A_{p1} in the tensile zone using the equation (18), where f_{pk} is the characteristic tensile strength of prestressing steel:

$$A_{p1} = \frac{P_t}{0{,}55 \cdot f_{pk}} \tag{18}$$

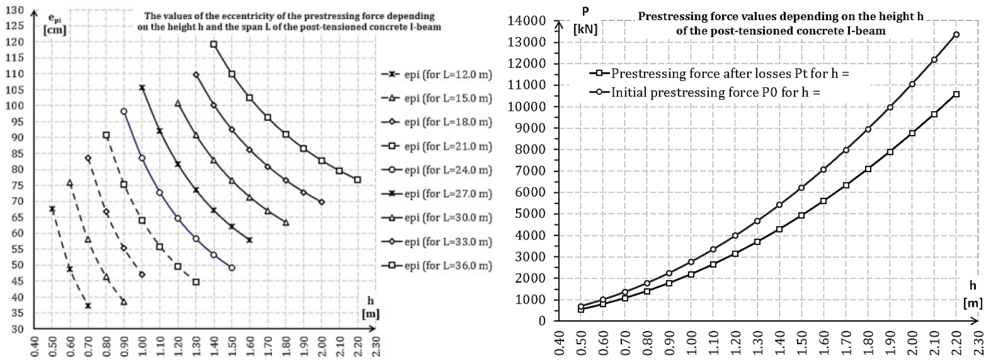


Fig. 4. Values of prestressing forces after all losses, values of initial prestressing forces (on the right diagram) and eccentricities of these forces (on the left diagram) as a function of cross-section height and beam span

4. Conclusions

The original graphic Magnel's method gives the possibility to determine the prestressing force and its eccentricity in a certain area (Fig. 3). The modification of this method presented in the article allows you to precisely determine the values of the prestressing forces and its eccentricity. It also allows you to quickly determine the above mentioned values for any span and height of the cross-section of the beam. It can be concluded that the presented modification of the Magnel method gives a precise solution to any post-tensioned concrete I-beam.

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Modyfikacja metody Magnela w projektowaniu przekrojów poprzecznych belek kablobetonowych

STRESZCZENIE:
Projektowanie przekrojów poprzecznych belek sprężonych z wykorzystaniem metody graficznej Magnela wprowadzono po raz pierwszy w połowie XX wieku. Metoda graficzna jest bardzo przydatna do wyznaczenia początkowej siły sprężającej oraz jej mimośrodu działania. Metoda ta jest stosowana również obecnie, pomimo różnych programów komputerowych przyspieszających proces wymiarowania konstrukcji sprężonych. W artykule przedstawiono w skrócie ideę metody Magnela, podjęto próbę powiązania klasycznego sposobu przyjmowania optymalnych wymiarów przekroju poprzecznego belki dwuteowej kablobetonowej z graficzną metodą Magnela wyznaczania siły sprężającej i jej mimośrodu. Celem tej próby jest uproszczenie metody graficznej Magnela i zastosowanie w obliczeniach geometrii analitycznej do precyzyjnego wyznaczenia optymalnej siły sprężającej oraz jej mimośrodu. Dokonano obliczeń sił sprężających i ich mimośrodów dla belek o rozpiętościach od 12,0 do 36,0 metrów, a wyniki przedstawiono na wykresach.

SŁOWA KLUCZOWE:
metoda Magnela; belki żelbetowe kablobetonowe; siły sprężające