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# New optimization algorithms and their application for 2D truss structures 

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#### Abstract

: In this study, two new algorithms named Rao-1 and Rao-2 are presented for the optimization of 2D truss structures. The main purpose of the optimization algorithms, used in this study, is to minimize the total weight of the truss structure. When carrying out this purpose, the allowable displacement and stress are taken into account as the constraints. The design variables are the cross-sectional areas of the steel truss bar elements. To calculate the structural response, the finite element analysis is coded in MATLAB. The optimal results obtained in this study are compared with those given in the literature in order to demonstrate the efficiency of the proposed algorithms.


## KEYWORDS:

optimization; Rao-1 and Rao-2 algorithm; 2D truss; structural analysis

## 1. Introduction

In recent years, researchers have proposed many optimizing algorithms. When proposing a new optimization algorithm, the main idea is to minimize computational time and find an effective solution in regards to related problems. In the case of civil engineering, these algorithms are applied as a solver for various different purposes, such as finding the minimum weight of a structure or the minimum or maximum values of frequency. As a classic example, the optimization of truss structures can be considered as a benchmark problem, refined by many studies using different algorithms.

Significant amounts of research can be found in the literature for the optimization of truss structures using different optimization algorithms. Dede and Ayvaz [1] used a teaching-learning-based algorithm (TLBO). They took into account the allowable nodal displacement and allowable stress constraints for tension and compression members. Grzywiński [2] presented a TLBO algorithm to shape and size truss optimization. Eskandar et al. [3] made a study on truss optimization by using a water cycle algorithm (WCA). They used discrete and continuous design variables for the size optimization of trusses. The topology optimization of truss structures is presented by Kaveh and Zolghadr [4] with a charged system search (CSS) by taking into account dynamic and static constraints. Cazacu and Grama [5] presented a study of size, shape and topology optimization of plane truss structures using a genetic algorithm (GA). The author of the paper implemented their proposed algorithm and finite element procedure in MATLAB. Using a harmony search algorithm (HS) another study was made by Lee and Geem [6] for 2D and 3D truss structures under multiple loading conditions with continuous design variables. Lamberti [7] prepared a study on truss, structures based on a simulated annealing algorithm

[^0](SA). He compared his optimal result with the result given in the literature based on a harmony search and particle swarm optimization (PSO) alogrithm. Tejani et al. [8] presented a study using four different metaheuristic optimization algorithms for planar and space truss structures. These optimization algorithms are an improved TLBO, improved heat transfer search (IHTS), improved water wave optimization (IWWO) and an improved passing vehicle search (IPVS). Joubari et al. [9] studied the size optimization of truss structures by taking into account the frequency constraints within a artificial bee colony algorithm (ABC). Grzywiński et al. [10] presented a study of a mid-size dome structure by using a Jaya algorithm (JA) with frequency constraints.

In this study, a new metheuristic algorithm developed by Rao [11] is examined for the optimization of 2D truss structures. To carry out the optimization process the computer codes for the optimization algorithm and the finite element analysis of the 2D truss structures were written in MATLAB. The allowable displacements for the free nodes and the allowable stress for tension and compression truss members are used as constraints. The design variables are the cross-sectional areas of the planar truss structure. The obtained optimal results were compared to results given in the literature using different optimization algorithms.

## 2. Definition of structural optimization of truss structures

For the size optimization of a truss structure, the minimum weight of the structure, which is the objective function of this optimization problem, is written by using the following equation:

$$
\begin{equation*}
W(x)=\sum_{i=1}^{n m} \rho * L_{i} * A_{i} \tag{1}
\end{equation*}
$$

where $W$ is the weight of the truss structure, $\rho$ is the density of the material, $L$ is the length of the truss member, $A$ is the cross-sectional area of the truss members and $n m$ is the number of truss members. If the design variables, which are the cross-sectional areas of the truss members, are categorized in a number of groupings, the cross-sectional areas for the same group members are the same value. In this case the Eq. (1) can be written in the following form:

$$
\begin{equation*}
W(x)=\sum_{i=1}^{n g} \rho * A_{i} * \sum_{j=1}^{n g m} L_{j} \tag{2}
\end{equation*}
$$

where $n g$ is the number of groups and $n g m$ is the number of group members for each grouping. The allowable displacement and the allowable stress should not be exceeded when the global optimal results are calculated. The constraints for the maximum displacement and the maximum stress are defined by the equations given below:

$$
\begin{array}{ccc}
\delta_{i} \leq \delta_{\text {all }}, & c_{i}=\max \left(\left|\frac{\delta_{i}}{\delta_{\text {all }}}-1\right|, 0\right) & i=1,2, \ldots, n n \\
\sigma_{j}^{t} \leq \sigma_{\text {all, }}^{t}, & c_{j}=\max \left(\left|\frac{\sigma_{j}^{t}}{\sigma_{\text {all }}^{t}}-1\right|, 0\right) & j=1,2, \ldots, n t m  \tag{3}\\
\sigma_{k}^{c} \leq \sigma_{\text {all }}^{c}, & c_{k}=\max \left(\left|\frac{\sigma_{j}^{c}}{\sigma_{\text {all }}^{c}}-1\right|, 0\right) & k=1,2, \ldots, n c m
\end{array}
$$

where $\sigma_{i}$ and $\sigma_{\text {all }}$ are the calculated and the allowable nodal displacement for the node $i, \sigma_{i}$ and $\sigma_{\text {all }}$ are the calculated and the allowable stress for the tensile or compression members, $n n, n t m$ and $n c m$ are the number of free nodes, number of tensile members and the number of compression members, respectively. To take into account the constraints of the structural optimization problem, a penalty function is calculated in terms of the violated constraints:

$$
\begin{gather*}
C=\sum_{i=1}^{n n} c_{i}+\sum_{j=1}^{n t m} c_{j}+\sum_{k=1}^{n c m} c_{k}  \tag{4}\\
f_{\text {penalized }}=W(x) *(1+C) \tag{5}
\end{gather*}
$$

At the beginning of the optimization process, the penalized objective function is generally greater than the objective function. But, at the end of the optimization process the penalized objective function must be equal to the objective function. In other words, the total constraint $C$ must equal to zero.

## 3. Optimization algorithms: Rao-1 and Rao-2

Similar to other population based metaheuristic algorithms, the Rao-1 (Eq. (6)) and Rao-2 (Eq. (7)) uses a randomly created initial population. Afterwards, the candidate solutions are modified in the hope that the newly modified solution will be better than the previous. If $X_{j, k, l}$ is the value of the $j^{\text {th }}$ variable for the $k^{\text {th }}$ candidate during the $i^{\text {th }}$ iteration, then this value is modified using (Eq. (6)) and (Eq. (7)):

$$
\begin{gather*}
X_{j, k, i}^{\prime}=X_{j, k, i}+r_{1, j, i}\left(X_{j, b e s t, i}-X_{j, \text { worst,i }}\right)  \tag{6}\\
X_{j, k, i}^{\prime}=X_{j, k, i}+r_{1, j, i}\left(X_{j, b e s t, i}-X_{j, w o r s t, i}\right)+r_{2, j, i}\left(\mid X_{j, k, i} \text { or } X_{j, l, i}|-| X_{j, l, i} \text { or } X_{j, k, i} \mid\right) \tag{7}
\end{gather*}
$$

Where, $X_{j, b e s t, I}$ and $X_{j, \text { worst }, I}$ are the best candidate and worst candidate for the variable $j$ in the current iteration $i . X_{j, k, l}^{\prime}$ is the updated value of $X_{j, k, l}$ and $r_{1, j l}$ and $r_{2, j, l}$ are random numbers in the range of $[0,1]$. The term $X_{j, k, I}$ or $X_{j, l, l}$ are two randomly selected candidates. Detailed information on these proposed algorithms can be found in reference [11]. A general flow chart for the proposed algorithms is given in Figure 1.


Fig. 1. The flow chart for the Rao-1 or Rao-2 algorithms

## 4. Numerical example

For a continuous 10-bar truss (Fig. 2), decision variables are from 0.1 to $35.0 \mathrm{in}^{2}$ (from 0.6452 to $225.806 \mathrm{~cm}^{2}$ ). The truss is subjected to a loading condition of P1 = $100 \mathrm{kips}(444.8 \mathrm{kN})$.

The material properties and the lower and the upper bounds of the cross-sectional areas of the design variables, which are the bar elements of the truss structures, are the same and are given in Table 1. The size of population and the maximum generation number are 20 and 2000, respectively.


Fig. 2. Ten-bar planar truss
Table 1
Structural constraints and material properties for 10-bar truss

| Symbols | Definitions | Value | Unit |
| :--- | :--- | :--- | :--- |
| $E$ | Modulus of elasticity | 10000 | ksi |
| $\rho$ | Material density | 0.1 | $\mathrm{lb}^{2} \mathrm{in}^{3}$ |
| $A$ | Cross-sectional area | $0.1 \leq \mathrm{A} \leq 35.0$ | $\mathrm{in}^{2}$ |
| $\delta$ | Allowable displacement | 2 for $x$ and $y$ direction | in |
| $\sigma$ | Allowable stress | $\pm 25$ | ksi |

A planar truss structure is selected to carried out the optimization process using the proposed algorithm. Optimization of a ten-bar planar truss structure was previously made by Lee and Geem [6], Renwai and Peng [12] and Eskandara et al. [3].

Table 2
Optimal results cross-section ( $\mathrm{cm}^{2}$ ) for 10-bar structure

| Design <br> variable | Eskandara <br> et al. [3] | Lee and <br> Geem [6] | Renwai and <br> Peng [12] | Rao-1 | Rao-2 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| A1 | 30.53 | 30.150 | 30.59 | 30.401 | 30.384 |
| A2 | 0.10 | 0.100 | 0.10 | 0.100 | 0.100 |
| A3 | 23.05 | 22.710 | 23.27 | 23.278 | 23.079 |
| A4 | 15.03 | 15.270 | 15.19 | 15.194 | 15.495 |
| A5 | 0.10 | 0.102 | 0.10 | 0.100 | 0.100 |
| A6 | 0.56 | 0.544 | 0.46 | 0.539 | 0.596 |
| A7 | 7.48 | 7.541 | 7.50 | 7.463 | 7.475 |
| A8 | 21.12 | 21.560 | 21.07 | 21.157 | 21.145 |
| A9 | 21.63 | 21.450 | $\mathbf{2 1 . 4 8}$ | 21.373 |  |
| A10 | 0.10 | $\mathbf{5 0 6 1 . 0 2}$ | $\mathbf{5 0 5 7 . 8 8}$ | $\mathbf{5 0 6 2 . 1 7}$ | $\mathbf{5 0 6 0 . 9 9}$ |
| Weight (kg) | $\mathbf{5 0 n}$ | $\mathbf{5 0 6 1 . 5 6}$ |  |  |  |

* this result violates constraints ( $\mathrm{C}=0.000907$ )

The optimal results obtained from this study using the proposed algorithms, Rao-1 and Rao-2 were compared with results given in previous studies in Table 2. As seen from this table, the best result is obtained from the Rao-1 algorithm. Although the optimal results given by Lee and Geem [6] is smaller than that of Rao-1, the result violates the constraints of the optimization problem.

## 5. Conclusions

The aim of this study was to make an optimal design for a planar truss structure using a new metaheuristic algorithm. To this purpose, a ten-bar planar structure was examined with both Rao-1 and Rao-2 algorithms and the results were compared the other algorithms found in the literature. For the finite element analysis of planar truss structures and the optimization algorithms the codes were written in MATLAB. It can be concluded that the new algorithm can be used in the design of planar truss structures.

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## Nowe algorytmy optymalizacji i ich zastosowanie do płaskiej kratownicy

## STRESZCZENIE:

Przedstawiono nowe algorytmy o nazwach Rao-1 i Rao-2 do optymalizacji płaskiej kratownicy. Głównym celem problemu optymalizacyjnego zastosowanego w tym badaniu jest minimalizacja całkowitej masy konstrukcji kratownicy. Przy realizacji tego celu dopuszczalne przemieszczenia i naprężenia są uwzględniane jako ograniczenia. Zmienne projektowe to pola przekroju poprzecznego stalowych elementów kratownicy. Aby obliczyć optymalną konstrukcję, zastosowano metodę elementów skończonych zakodowaną w MATLAB-ie. Optymalne wyniki uzyskane w tym badaniu są porównywane z podanymi w literaturze w celu wykazania wydajności proponowanego algorytmu.

SŁOWA KLUCZOWE:
optymalizacja; algorytm Rao-1 i Rao-2; kratownica płaska; analiza konstrukcji


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