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Formulation of the critical lateral buckling moment of a steel I-beam

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ABSTRACT:
The paper presents an analysis of the method of formulation of the critical lateral moment during buckling of an I-beam. Three methods of determining the critical moment were considered: commonly known analytical formulas, the Robot Structural Analysis Professional 2024 program and the finite element method implemented in the ADINA program. The subject of the analysis was the steel beam with a cross-section of IPE 200. Two simply supported beams were considered. The first one was loaded with a concentrated force and the second one was loaded with an evenly distributed load.

KEYWORDS:
I-beam; buckling; FEM; critical lateral bending moment

1. Introduction

Lateral torsional buckling may occur in an unrestrained beam. The beam is considered to be unrestrained when its compression flange is free to displace laterally and rotate. When an applied load causes both lateral displacement and twisting of a member, lateral torsional buckling has occurred [1]. Cross-sections for which the moments of inertia differ significantly $I_y >> I_z$ and have relatively low torsional stiffness, when loaded in a plane with greater bending stiffness, are bent relative to the weaker axis, this is the loss of the flat form of bending, which is accompanied by twisting relative to the longitudinal axis [2].

The bending beam may lose the general or local stability of the web or compression flange depending on the type and method of loading, the shape and geometric features of the cross-section, length, support conditions or indirect elastic constraints.

Lateral torsional buckling, where the behaviour changes from mainly in-plane bending to combined lateral deflection and twisting, is one of the most important stability problems and may often be a controlling factor in steel beam design. Therefore, various design standards and codes recommend methods in order to calculate lateral torsional buckling of steel members.

In paper [3], the explanation of the elastic critical moment M_{cr} for doubly symmetric cross-sections applied in Eurocode 3 [4] was presented. The explanation of the elastic critical moment M_{cr} was also presented in the Access Steel guide [5]. A comparison of the critical moment determined using the formulas and the experimental results was presented in [1]. The works [6, 7] presented a comparison of the critical moment determined using formulas and the finite element method. As presented in [7], critical moments estimated with formulas provided sufficient engineering approximation when compared with the values obtained with the FEM software. However,

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authors of paper [6] emphasize that the analytical calculation path can only be applied to systems of low complexity and the use of FEM has a significant advantage.

2. Materials and methods

The aim of the study was an analysis of the method of formulation the critical lateral moment during buckling of an I-beam. Three methods of determining the critical bending moment were considered: commonly known analytical formulas, the Robot Structural Analysis Professional program and the finite element method implemented in the ADINA program.

The subject of the research was the steel beam with the cross-section of IPE 200 made of S235 steel grade [8]. Two simply supported beams were considered. The first one was loaded with a concentrated force and the second one was loaded with an evenly distributed load.

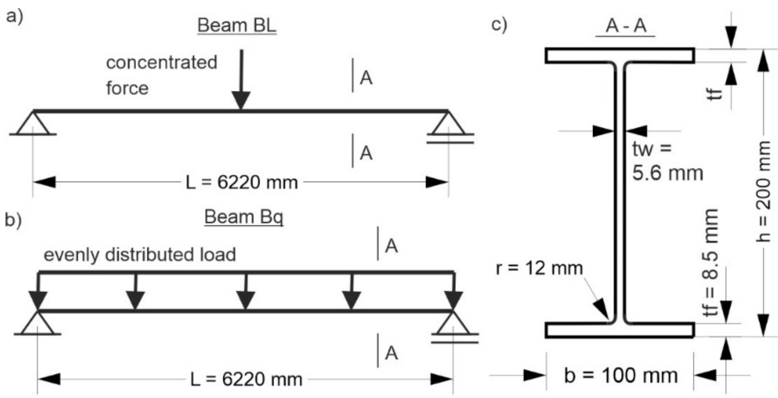

Fig. 1. Analysed beams: a) loaded with concentrated force, b) loaded with evenly distributed load, c) cross-section shape

2.1. Calculations based on formula

The calculations were performed based on the formula dedicated for the beam with a common case of normal support conditions at the ends (fork supports) [5], as follows:

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L^2} \left(\sqrt{\frac{I_\omega}{I_z} + \frac{L^2 G I_t}{\pi^2 E I_z} + (C_1 z_g)^2} - C_2 z_g \right) \tag{1}$$

where:
E is the Young's modulus,
G is the shear modulus,
I_z is the second moment of area about the weak axis,
I_t is the torsion constant,
I_ω is the warping constant,
L is the beam length between points which have lateral resistant,
z_g is the distance between the point of load application and the shear centre,
C_1 and C_2 are the coefficients depending on the load and end restraint conditions.

In the calculation carried out for beam BL loaded by concentrated force C_1 and C_2 coefficients were assumed as 1.348 and 0.630 respectively, but for beam Bq loaded by distributed load, C_1 and C_2 coefficients were equal 1.127 and 0.454.

The position of load on the upper flange were assumed in the calculations.

2.2. Robot Structural Analysis Professional

In the Robot Structural Analysis Professional program, the beam was modelled using the 2D rod with the cross-section of IPE 200. The S235 steel grade was applied for the beam structure with Young's modulus and shear modulus equal to 210 GPa and 81 GPa. As presented in Figure 2, the length of the beam was 6220 mm.

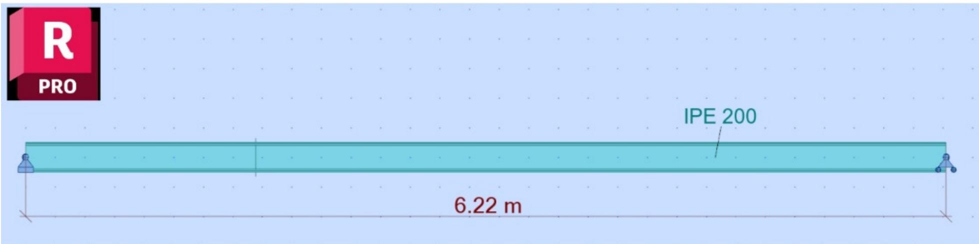

Fig. 2. Beam analysed in Robot Structural Analysis Professional program

Assumed boundary conditions reflected the freely supported beam. Both types of loads were applied to the beams. The position of load on the upper flange and general distribution on internal forces were assumed in the program options.

2.3. Finite element method

The numerical model was built with 8-node 3D-solid elements. The interaction between finite elements was achieved by connections of those common nodes. The boundary conditions were assumed at the surface cross-section at the ends of the beam (Fig. 3). Both types of loads were applied to the beams as loads equal to 1.

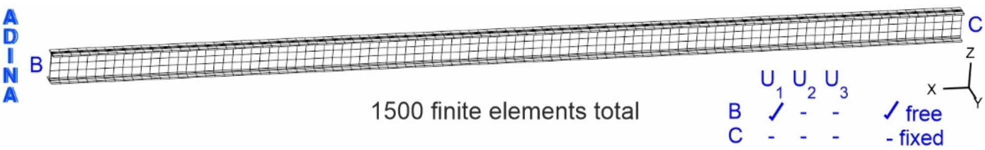

Fig. 3. Beam analysed in ADINA program

The bilinear-plastic material model with Young's modulus of 210 GPa were used.

3. Results

The calculation of the critical lateral moment for the BL beam loaded with concentrated load was performed as follows:

$$M_{cr} = 1.348 \frac{\pi^2 \cdot 210\ \text{GPa} \cdot 142\ \text{cm}^4}{(6220\ \text{mm})^2} \cdot \left(\sqrt{\frac{12980\ \text{cm}^6}{142\ \text{cm}^4} + \frac{(6220\ \text{mm})^2 \cdot 81\ \text{GPa} \cdot 7\ \text{cm}^4}{\pi^2 \cdot 210\ \text{GPa} \cdot 142\ \text{cm}^4} + (0.630 \cdot 100\ \text{mm})^2} - 0.630 \cdot 100\ \text{mm} \right) = 23.90\ \text{kNm} \tag{2}$$

The calculation of the critical lateral moment for the Bq beam loaded with an evenly distributed load was performed as follows:

$$M_{cr} = 1.127\frac{\pi^2 \cdot 210\ \mathrm{GPa} \cdot 142\ \mathrm{cm}^4}{(6220\ \mathrm{mm})^2} \cdot \left(\sqrt{\frac{12980\ \mathrm{cm}^6}{142\ \mathrm{cm}^4} + \frac{(6220\ \mathrm{mm})^2 \cdot 81G\ \mathrm{Pa} \cdot 7\ \mathrm{cm}^4}{\pi^2 \cdot 210\ \mathrm{GPa} \cdot 142\ \mathrm{cm}^4} + (0.454 \cdot 100\ \mathrm{mm})^2} - 0.454 \cdot 100\ \mathrm{mm}\right) = 21.21\ \mathrm{kNm} \tag{3}$$

The results of critical lateral bending moments obtained by the Robot Structural Analysis and ADINA programs were presented in Table 1. For both beams BL and Bq the loads corresponded to the critical lateral moment were also presented in Table 1.

Additionally, the result of the buckling beam obtained in ADINA program was presented in Figure 4. It is easy to see, that on the whole length of the beam, the cross-section rotates around the X axis. Both under the concentrated and distributed load, the way of buckling of the beam was the same.

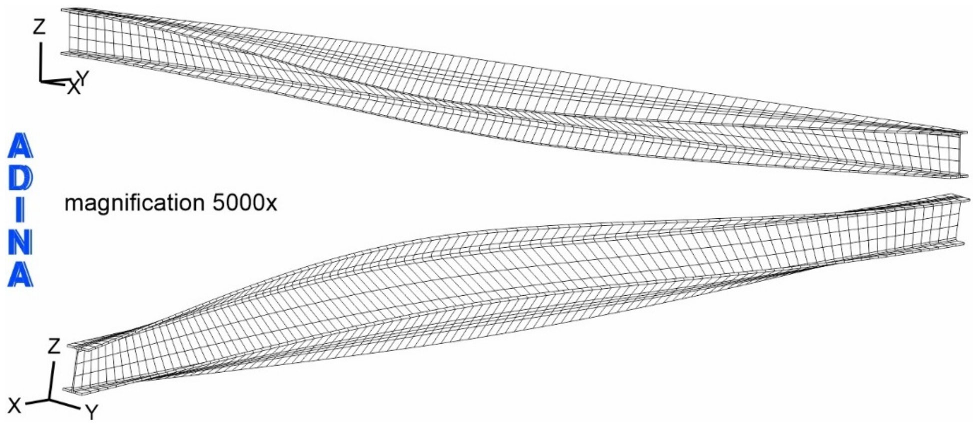

Fig. 4. Beam under the buckling state

Considering the results presented in Table 1, for the BL beam with concentrated load, the critical lateral moment obtained by the Robot Structural Analysis and ADINA are similar. The difference is only about 3 %. But the critical lateral moment calculated from the formula is equal to 23.90 kNm and it is about 57 % the value from the Robot Structural Analysis program.

Considering the Bq beam with distributed load, the critical lateral moment obtained by the Robot Structural Analysis is equal to 29.21 kNm which is 69 % of results from the ADINA program (42.30 kNm). The critical lateral moment calculated from the formula is even smaller and is equal to 21.21 kNm. So, it is 73 % and 50 % of the value resulted from the Robot Structural Analysis and ADINA programs, respectively.

Table 1
Results of critical lateral bending moment and corresponded loads

Beam	Formula		Robot Structural Analysis		ADINA	
	Critical moment [kNm]	Load	Critical moment [kNm]	Load	Critical moment [kNm]	Load
BL	23.90	15.37 kN	41.32	26.57 kN	42.30	27.20 kN
Bq	21.21	4.39 kN/m	29.21	6.04 kN/m	42.30	8.75 kN/m

It is worth noting that the critical lateral bending moments obtained from the numerical simulations are the same for both beams, regardless of the type of load applied. Comparing the results from Robot Structural Analysis, the critical moment for the beam under the distributed load is 41 % less than the critical moment for the beam under the concentrated load. Comparing the results of calculations from the formula, the critical moment for the beam under the distributed load is 12 % less than critical moment for the beam under the concentrated load.

The differences between the ways of formulating the critical lateral moment are visible and dependent on a few factors.

4. Discussion

Regardless of the type of load, the values of the critical lateral moment calculated by the formula are the smallest among the considered methods. This is due to the approximations used resulting from the following assumptions: relatively simple calculations and obtaining a safe result, i.e. with a lower value compared to the results from some experiments. At the same time, the formulas take into account the possibility of geometric and material imperfections, which are included in the values of the C_1 and C_2 coefficients. Formulas obtained through strict theoretical considerations work well for simple cases, but for components with more complicated support conditions and unusual bending moment distributions, standard formulas may turn out to be insufficient and the results obtained from them unreliable.

The Robot Structural Analysis program uses the finite element method to determine internal forces. Robot Structural Analysis conducts further calculations in accordance with the formulas included in the Eurocode 3 standard. Therefore, in the analysed case, the C_1 and C_2 coefficients determined by the program were calculated based on the bending moment diagrams. Then Robot Structural Analysis used the coefficient values to calculate the critical moment according to formula (1). Thanks to this, the C_1 and C_2 coefficients were determined more precisely for specific cases, while the next stage was identical with calculations in accordance with the formulas.

The ADINA program is fully based on the finite element method. Therefore, regardless of the method of loading the beam, the values of critical moments were identical. At the same time, these values were the largest among the analysed methods. The differences in the results obtained using simulation in relation to other methods resulted, among others, from differences between the values of the geometric characteristics of the cross-section. In a numerical model built from 3D-solid elements, it is not possible to define cross-sectional characteristics, e.g. consistent with profile tables. At the same time, the difference in the critical moment obtained using numerical simulations and calculations using the Robot Structural Analysis program is caused by using the different finite elements and the different way of setting boundary conditions (to the points in the Robot Structural Analysis program and to the surfaces in the numerical model). It should also be added that the numerical simulations did not take into account geometric or material imperfections. Nevertheless, it is a reliable method for determining the critical lateral moment for complex structures with unusual geometries and uneven distribution of bending moment.

Due to the discrepancy in the results obtained using the three methods, in the future the experimental tests determining the critical lateral buckling moment and comparison of its value with the calculation methods analysed in this paper are planning to perform.

5. Conclusions

- The values of critical lateral bending moment calculated by the formula are about 40 % less than results of numerical simulations.
- Formulas for critical lateral moment obtained through theoretical considerations work well for simple cases, but for components with more complicated support conditions and an unusual bending moment, standard formulas may turn out to be insufficient.

– Numerical simulation is the reliable method for determining the critical lateral moment for complex structures with unusual geometries and uneven distribution of bending moment.
– The Robot Structural Analysis program can be used to access the critical lateral moment, especially when simulating a complex civil engineering structure with Eurocode requirements, but the results are underestimated compared to calculations using totally FEM model.

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Obliczenie krytycznego momentu zginającego stalowej belki dwuteowej

STRESZCZENIE:
Przedstawiono analizę sposobu obliczania momentu krytycznego przy zwichrzeniu belki dwuteowej. Rozważano trzy sposoby wyznaczania momentu krytycznego: powszechnie znane wzory analityczne, program Robot Structural Analysis Professional 2024 oraz metodę elementów skończonych zaimplementowaną w programie ADINA. Przedmiotem analizy była belka stalowa o przekroju IPE 200. Rozważano dwie belki swobodnie podparte. Pierwszą obciążono siłą skupioną, a drugą obciążono obciążeniem równomiernie rozłożonym.

SŁOWA KLUCZOWE:
belka; wyboczenie; MES