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DETERMINISTIC AND PROBABILISTIC ANALYSIS OF THE RANDOM TORSIONAL EFFECTS OF THE SEISMIC RESISTANCE OF SYMMETRICAL TALL BUILDINGS

Introduction

Recent advances and the general accessibility of information technologies and computing techniques give rise to assumptions concerning the wider use of the probabilistic assessment of the reliability of structures through the use of simulation methods [1-5]. Much attention should be paid to using the probabilistic approach in an analysis of the reliability of structures [6, 7].

Most problems concerning the reliability of building structures are defined today as a comparison of two stochastic values, loading effects *E* and the resistance *R*, depending on the variable material and geometric characteristics of the structural element.

The deterministic definition of the reliability condition has the form

$$R_d \geq E_d \tag{1}$$

and in the case of the probabilistic approach, it has the form

$$RF = R - E \geq 0 \tag{2}$$

where *RF* is the reliability function.

The most general form of the probabilistic reliability condition is given as follows:

$$p_f = P(R - E < 0) \equiv P(RF < 0) < p_d \tag{3}$$

where p_d is the so-called design ("allowed" or "acceptable") value of the probability of failure [8]. From the analytic formulation of the probability density by the

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functions $f_R(r)$ and $f_E(e)$ and the corresponding distribution functions $\Phi_R(x)$ and $\Phi_E(x)$, the probability of failure can be defined in the general form:

$$p_f = \int_{-\infty}^{\infty} dp_f = \int_{-\infty}^{\infty} f_E(x)\Phi_R(x)dx = \int_{-\infty}^{\infty} \Phi_E(x) f_R(x)dx \tag{4}$$

This integral can be solved analytically only for simple cases; in a general case it should be solved using numerical integration methods after discretization.

1. Analysis of the building earthquake resistance

The seismic resistance of the core-column structural system was investigated using the deterministic and probabilistic analysis. The Eurocode EN 1998-1 [9] requires, that the seismic resistance of structures must be checked to the accidental torsion effects.

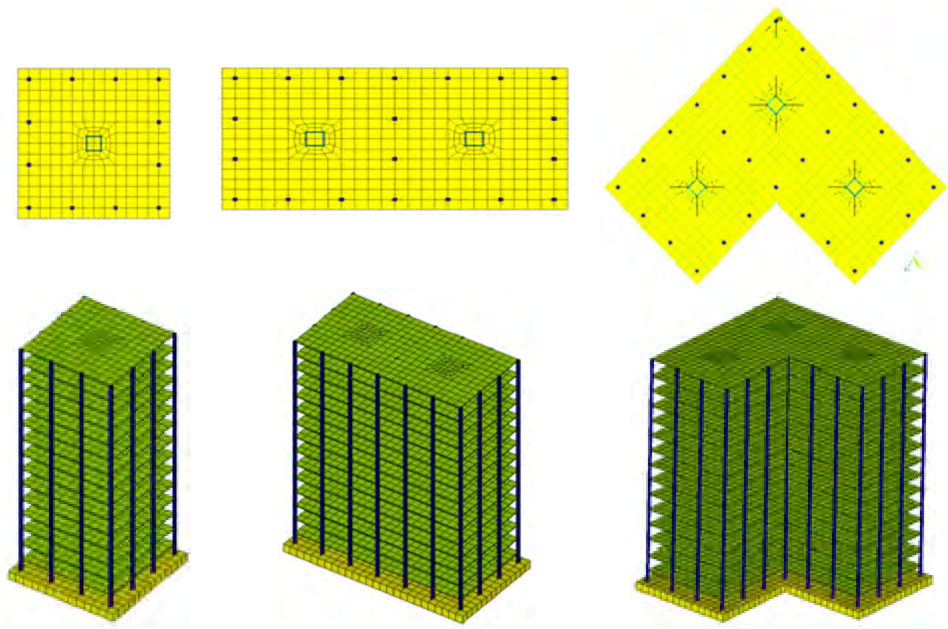

Fig. 1. Model VSB1 - bisymmetrical, VSB2 - one symmetrical and VSB3 - L symmetrical

In order to cover uncertainties in the location of masses and in the spatial variation of the seismic motion, the calculated center of mass at each floor i shall be considered displaced from its nominal location in each direction by an accidental eccentricity due to the requirements of EN 1998-1 [9]. The accidental torsion effect is defined as follow

$$e_{ai} = \pm 0.05 \cdot L_i \tag{5}$$

where e_{ai} is an accidental eccentricity of storey mass i from its nominal location, applied in the same direction at all floors, L_i is a floor-dimension perpendicular to the direction of the seismic action.

The influence of accidental torsion effect will be considered using the accidental eccentricity of storey mass for deterministic analysis. In the case of probabilistic analysis this effect should be considered as variable nonsymmetrical distribution of masses.

The neglecting of the random eccentricities of the mass distribution or seismic excitation in the calculation of the seismic response of the symmetric buildings could be the reason of the structure failure in the reality.

The seismic analysis of the nonsymmetrical masses distribution is considered on two types of tall building - one is the biaxial symmetric structures with one central core and columns and second is the symmetrical structures with two cores and columns. A 15 storey buildings with the prismatic, square cross-section and constant storey height of 3 m, shown in Figure 1 are considered.

All columns in these buildings are 500/500 mm in cross-section. The outer dimensions of the cores are 2 x 2 m with the wall thickness of 150 mm. A reinforced concrete foundation plate with thickness 1 m has 21 x 21 m and 21 x 39 m in horizontal plane. The thickness of floor reinforced concrete plate is 220 mm. All floor slabs have a permanent load of 1.5 kN/m^2 and variable load of 3.0 kN/m^2. The material properties of this concrete building are Young's modulus, $E = 30\,000$ MPa and Poisson's ratio $\mu = 0.2$.

2. Loading and load combination

The loading and load combination in the case of the deterministic as well as the probability calculation is different due to requirements of EN 1990 [8] and JCSS 2000 [10], too.

The seismic load was taken in accordancewith EN 1998-1 [9] as a design acceleration response spectrum for B type of soil and design acceleration $a_g = 1.0$ ms^{-2}.

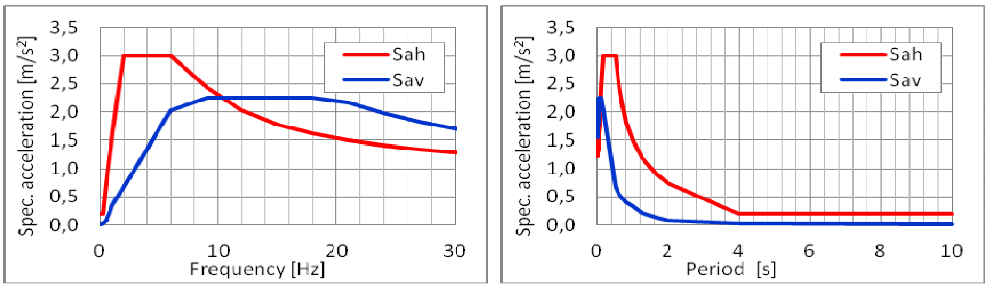

Fig. 2. Response spectrum acceleration in horizontal and vertical direction

In the case of deterministic calculation and the ultimate limit state of the structure the load combination is considered according to EN 1990 [8] as follows:

➲ *Seismic design situation - deterministic method*

$$E_d = G_k + \psi_{2.Q} Q_k \pm A_{Ed} \tag{6}$$

where G_k is the characteristic value of the permanent loads, Q_k - the characteristic value of the variable loading, A_{Ed} ($=\gamma_I A_{Ed.k}$) - the design value of the seismic loading, $A_{Ed.k}$ - the characteristic design value of the seismic loading, γ_I - importance factor (of the building structure), $\psi_{2.Q}$ - the combination factor according to EN 1990 ($\psi_{2.Q}$ = 0.3).

In the case of probabilistic calculation and the ultimate limit state of the structure the load combination we take following:

➲ *Seismic design situation - probability method*

$$E = G + Q + A_E = g_{\mathrm{var}} G_k + q_{\mathrm{var}} Q_k + a_{\mathrm{var}} A_{E.k} \tag{7}$$

where g_{var}, q_{var}, a_{var} are the variable parameters defined in the form of the histogram calibrated to the load combination in compliance with Eurocode [8].

3. Spectrum analysis of the building seismic resistance

The seismic analysis of the tall building structure was realized using the linearized response spectrum method [4]. This method allows an approximate determination of the maximum response of an MDOF system without performing a time history analysis. The response spectrum method is based on the solution of dynamic equation by modal superposition method in time.

The dynamic equation for MDOF system with n-DOF due to support excitation is defined in the form

$$\mathbf{M}(\ddot{\mathbf{u}} + \ddot{\mathbf{u}}_s) + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\,\mathbf{u} = \mathbf{0} \tag{8}$$

where **M**, **C**, **K** are matrix (n x n) of the mass, damping and stiffness, $\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}$ are vectors (n x 1) of relative displacements, velocities and accelerations, $\ddot{\mathbf{u}}_s$ is vector (n x 1) of support accelerations (seismic excitation). After transformation the equations (8) to the modal coordinate system by next substitution

$$\mathbf{u} = \sum_{i=1}^{m} \mathbf{\Phi}_i . \mathbf{Y}_i \tag{9}$$

we obtain the m-independent equations of motion in the form

$$\ddot{\mathbf{Y}}_i + 2\xi_i\,\omega_i\,\dot{\mathbf{Y}}_i + \omega_i^2\,\mathbf{Y}_i = -\mathbf{\Gamma}_i\,\ddot{\mathbf{u}}_s \tag{10}$$

where $\mathbf{\Phi}_i$ is an eigenvector (m x 1) for mode i after normalization of mass matrix $\mathbf{\Phi}_i^T \mathbf{M} \mathbf{\Phi}_i = 1$, $\mathbf{Y}_i$ is a modal coordinate vector (m x 1), ξ_i is relative damping for mode i, ω_i angular frequency mode i, $\mathbf{\Gamma}_i$ is participation factor for mode i in the form

$$\Gamma_i = \frac{\Phi_i^T \mathbf{M1}}{\Phi_i^T \mathbf{M} \Phi_i} \tag{11}$$

The response spectrum of the displacements and forces from the excitation in direction $a = 1, 2, 3$ is calculated from the modal response by method square root of sum of squares mode (SRSS) in the form

$$R_a = \sqrt{\sum_{i=1}^{N} (R_i)^2} \tag{12}$$

The total response spectrum is calculated from three base acceleration spectra (in space) alternatively from the combination SRSS or standard combination rule

$$R_{tot} = R_1 + 0.3R_2 + 0.3R_3 \quad \text{or} \quad R_{tot} = 0.3R_1 + 0.3R_2 + R_3 \quad \text{or} \quad R_{tot} = 0.3R_1 + R_2 + 0.3R_3 \tag{13}$$

where R_i (i = 1, 2, 3) are response values from the acceleration excitation in the direction 1, 2, 3.

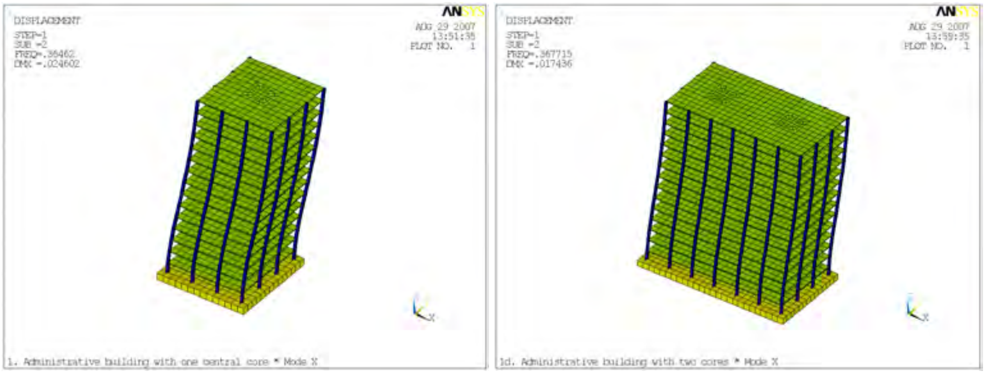

Fig. 3. First mode of the building with one central core - model VSB1 and VSB2

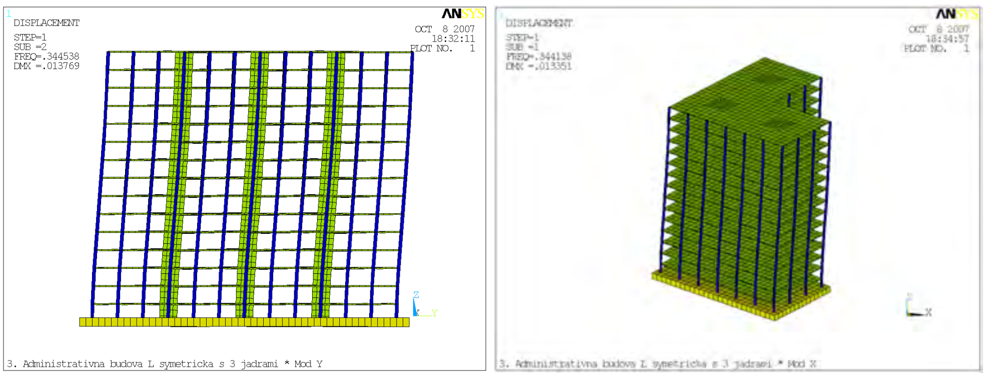

Fig. 4. First mode of the building with three cores - model VSB3

The modal analysis of these two models of buildings was realized on the software ANSYS using Lanczos method.

4. Uncertainties of input variables

The effect of soil-structure interaction can be investigated in the case of probabilistic assessment by sensitivity analysis of the influence of variable properties of soil. A soil stiffness variability in the vertical direction is defined by the characteristic stiffness value k_z from the geological measurement and the variable factor $k_{z.\mathrm{var}}$. The stiffness of the structure is determined with the characteristic value of Young's modulus E_k and variable factor e_{var}. A load is taken with characteristic values G_{k}, Q_{k}, $A_{E,k}$ and variable factors g_{var}, q_{var} and a_{var}. The uncertainties of the calculation model are considered by variable model factor θ_R and variable load factor θ_E for Gauss normal distribution.

$$q(x,y)=\left\{q_{z,\mathrm{var}}+2q_{yy.\mathrm{var}}\left[(x-x_o)/l_x\right]+2q_{xx.\mathrm{var}}\left[(y-y_o)/_y\right]\right\}q_k \tag{14}$$

where q_k is characteristic value of live load (q_{k} = 1 kN/m^2), x_o, y_o are coordinates of building slab gravity centre, L_x and L_y are the plane dimensions of the slabs in directions x and y.

The random live load is considered in two alternatives - model VSB1p takes the variable live load on the plane only and model VSB1pm takes the variable live load both on the plane and height of building. The variability of the stiffness and masses parameters of the building has the significant influence on the modal characteristic of the building.

TABLE 1

Probabilistic model of input parameters

Name	Quantity	Charact. value	Variable paramet.	Histogram	Mean	Stand. deviation	Min. value	Max. value
Soil	Stiffness	$k_{z,k}$	$k_{z\cdot\mathrm{var}}$	Normal	1	0.200	0.451	1.490
Material	Young's modulus	E_k	e_{var}	Normal	1	0.120	0.645	1.293
Load	Dead	G_k	g_{var}	Normal	1	0.010	0.755	1.282
			$q_{z\cdot\mathrm{var}}$	Uniform	0.5	0.290	0	1
	Live	Q_{k1}	$q_{xx\cdot\mathrm{var}}$	Uniform	0	0.580	–1	1
			$q_{yy\cdot\mathrm{var}}$	Uniform	0	0.580	–1	1
	Seismic	$A_{Ed,k}$	a_{var}	Beta (T. I)	0.67	0.142	0.419	1.032
Resistance	Shear	Vu_k	$V_{u.\mathrm{var}}$	Lognormal	1	0.100	0.726	1.325
Model	Action uncert	θ_E	$T_{e.\mathrm{var}}$	Normal	1	0.100	0.875	1.135
	Resist. uncert	θ_R	$T_{r.\mathrm{var}}$	Normal	1	0.100	0.875	1.135

The results of the probability analysis of the all models present that the principal frequencies in the direction X (resp. Z) are in the interval from 0.32 to 0.48 Hz (resp. from 2.77 to 6.00 Hz). This frequency interval has the important influence to intensity value of seismic excitation on the part of the response spectrum acceleration about of frequency 2 Hz (resp. 7 Hz).

5. Reliability criteria

Reliability of the structures is designed in accordance of standard requirements STN 731201 and EN 1998-1 [9] for ultimate and serviceability limit state.

Damage limitation of the reinforced concrete structures depend on the criterion of the maximum interstorey drifts. The standard EN 1998-1 [9] define the function of failure in the form

$$g(d) = 1 - d_E / d_R \geq 0 \tag{15}$$

where d_E is interstorey horizontal displacement, d_R ($d_R = 0.005\, h / \nu$) is limit value of interstorey horizontal displacement defined (for non-structural elements of brittle materials attached to the structure), h is storey height (h = 3 m) and ν is reduction factor to take into account the lower return period of the seismic action associated with the damage limitation requirement (ν = 0.4).

6. Sensitivity analysis

Sensitivity analysis of the influence of the variable input parameters to the reliability of the structures depends on the statistical independency between input and output parameters [4]. Matrix of correlation coefficients of the input and output parameters is defined by Spearman

$$r_s = \frac{\sum_i^n \left(E_i - \bar{E}\right)\left(R_i - \bar{R}\right)}{\sqrt{\sum_i^n \left(E_i - \bar{E}\right)^2} \sqrt{\sum_i^n \left(R_i - \bar{R}\right)^2}} \tag{16}$$

where E_i is rank of input parameters within the set of observations $[x_i]^T$, R_i is rank of output parameters within the set of observations $[y_i]^T$, $\bar{E}, \bar{R}$ are average ranks of the parameters E_i and R_i respectively.

The results of the sensitivity analysis of the interstorey drift present that the variability of seismic load has the most significant influence on the interstorey drift in the model VSB1p, VSB1pm and VSB3p too. A variability of the live load is second important parameter in the model VSB3p.

The variability of dead load and material stiffness (e.g. live load and dead load) are significant for first mode in model VSB1p (e.g. VSB3p). In the case of the shear resistance sensitivity analysis of the building, the most important variable parameters in the model VSB1p are the seismic load, material stiffness, live load in the vertical direction. Otherwise the seismic load and live load on the model VSB3p are the variables with significant effect. The sensitivity analysis gives the valuable information about the influence of uncertainties of input variables (load, material and model) to engineer for optimal design of the structures.

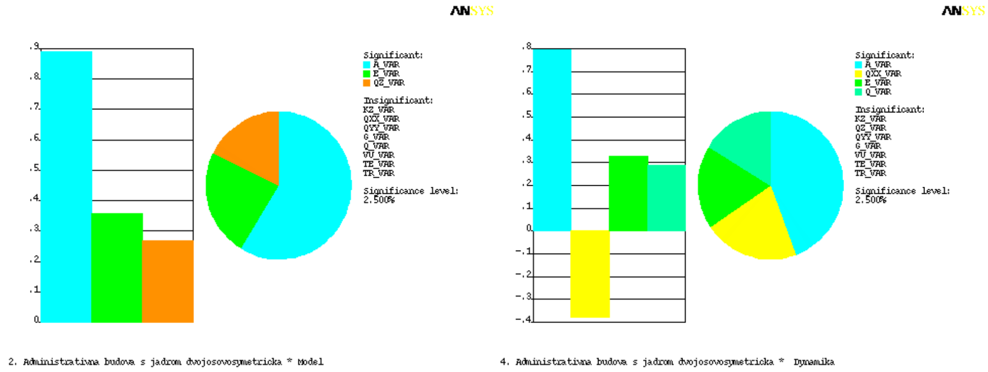

Fig. 5. Sensitivity analysis for the shears resistance in the model VSB1p and VSB3p

TABLE 2

Comparison of deterministic and probabilistic analyses

Method	Model	Extreme interstorey drift [mm] $d_r = 37.5$ mm				Extreme shear force [kN] $V_R = 1990$ kN/m			
		Min	Max	Mean	St. dev.	Min	Max	Mean	St. dev.
Model of tall building with one reinforced concrete core - VSB1									
Deterministic	VSB1d	–	15.72	–	–	–	1009.5	–	–
	VSB1de	–	26.11	–	–	–	1349.2	–	–
Probabilistic	VSB1p	7.73	22.15	13.56	3.99	710.15	2356.0	1316.3	321.2
Model of tall building with two reinforced concrete cores - VSB2									
Deterministic	VSB2d	–	18.19	–	–	–	841.2	–	–
	VSB2de	–	31.72	–	–	–	1416.7	–	–
Probabilistic	VSB2p	8.34	28.84	15.21	3.72	712.36	2481.6	1364.8	339.5
Model of tall building with three reinforced concrete cores - VSB3									
Deterministic	VSB3d	–	9.39	–	–	–	517.2	–	–
	VSB3de	–	35.37	–	–	–	1537.8	–	–
Probabilistic	VSB3p	4.16	14.45	7.49	1.98	429.4	1514.0	758.3	211.8

Deterministic calculation of the seismic resistance of the tall building was taken on the model VSB1d, VSB1de, VSB2d, VSB2de and VSB3d, VSB3de. The probabilistic calculation was carried out on the model VSB1p, VSB2p and VSB3p. The influence of the eccentricity of masses center was considered on the models with index "e". The comparison of deterministic and probabilistic analyses is presented in Table 2. The influence of the torsional eccentricity to the interstorey drift in deterministic (or probabilistic) analysis is equal to 66% (or 41%) in VSB1d (or VSB1p) model, 74% (or 59%) in VSB2d (or VSB2p) model and 276% (or 54%) in VSB3d (or VSB3p) model. The lower differences are in the extreme shear forces.

Conclusions

The influence of the accidental torsional effect to the seismic resistance of the reinforced concrete tall buildings with the core-columns system was investigated in this paper in accordance with the Eurocode requirements. This effect represents the uncertainties in the location of masses and in the spatial variation of the seismic motion. The methodology of the seismic analysis of the reinforced concrete tall building structures on the base of deterministic and probabilistic assessment was considered. This analysis was realized on the example of the one, biaxial and *L*-symmetrical tall building structures. In the case of deterministic seismic analysis the accidental eccentricity of masses is defined in the Eurocode. The random distribution of mass eccentricity on the building slab was considered as the linear approximation of the distribution of live load on all the floors (VSB1p, VSB2p, VSB3p). From the summary of the results it follows that the unequal distribution of the masses has the significant influence on the structure seismic resistance. The maximum difference in the interstorey drift was obtained in the building *L*-symmetrically with 3 cores (model VSB3). The deterministic analysis with the eccentricity of masses give us more conservative results as probabilistic analysis.

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Abstract

This paper presents the results from the deterministic and probabilistic seismic analysis of the influence of center mass eccentricity to symmetric tall buildings. During the structural design process, the problems of the safety, reliability and durability of a single structural element as well as the entire structure from the point of view of its planned life cycle must be considered. The methodology of the seismic analysis of the reinforced concrete tall building structures on the base of deterministic and probabilistic assessment is presented. The possibilities of the utilization the LHS method to analyze the extensive and robust tasks in FEM is presented in the case of tall buildings with central core and columns system. The analysis of the seismic resistance of the structure was calculated in the ANSYS program.

Keywords: earthquake, accidental torsion, building, probability, Eurocode, FEM, LHS

Analiza deterministyczna i probabilistyczna efektów przypadkowego skręcenia odpornych na działania sejsmiczne symetrycznych budynków wysokich

Streszczenie

Przedstawiono wyniki deterministycznej i probabilistycznej analizy sejsmicznej wpływu ekscentryczności masy środkowej na symetryczne budynki wysokie. Podczas projektowania strukturalnego należy wziąć pod uwagę problemy bezpieczeństwa, niezawodności i trwałości pojedynczego elementu konstrukcyjnego, a także całej konstrukcji z punktu widzenia planowanego cyklu eksploatacji. Przedstawiono metodologię analizy sejsmicznej zbrojonych konstrukcji żelbetowych na podstawie oceny deterministycznej i probabilistycznej. Możliwości wykorzystania metody LHS do analizy rozbudowanych zadań w MES przedstawiono dla przypadków wysokich budynków z centralnym systemem rdzeni i kolumn. Analizę odporności sejsmicznej konstrukcji wykonano w programie ANSYS.

Słowa kluczowe: trzęsienie ziemi, przypadkowe skręcanie, budynek, prawdopodobieństwo, Eurokod, MES, LHS